Quin Buob
HW 4

1) a) $t=0$ $A(t)=0$, $T(t)=50$
It takes A $50/200 = 1/4$ hr to get T's starting point at which time T made it $50+50(\frac{1}{4}) = 62.5$ so
$t = 1/4$ $A(t)=50$, $T(t)=62.5$
At $t=\frac{1}{4}+\frac{1}{16}=5/16$ $A(t)=62.5$, $T(t)=65.625$
Continueing in this same way
$t=21/64$ $A(t)=65.625$ $T(t)=66.40625$
So on and soforth, Whenever A will reach the point T was in the previus step, T would have gotten a bit further, ad infinitum, so A will never catch T

b) $A(t)=200t$ $T(t)=50t+50$

$$200t = 50t+50$$
$$150t=50$$
$$t = 1/3 \text{ hr}$$

A will reach t at 1/3 of an hr or 20 min

c) Adding succesive time intervals

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}\dots = \frac{1}{4}\left(1+\frac{1}{4}+\frac{1}{16}+\dots\right) = \frac{1}{4}\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$$

B/c this is a geometric series it converges to

$$\frac{1}{4} \times \left(\frac{1}{1-1/4}\right) = 1/3 \text{ hr}$$

The sum of the geometric series is the same a (b)

d) $t = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = 21/64$ $A(t) = 65.625$ $T(t) = 66.40625$

The next step requires $1/256$ which is meaningless if time can only advance at $1/64$ hrs

There fore the next step

$t = 21/64 + 1/64 = 22/64$

$A(t) = 68.75$ $T(t) = 67.1875$

A has caught up and passed T, there fore time must be discrete

2) Prove

$$1 + x + x^2 + \dots x^n = \frac{1-x^{n+1}}{1-x}$$

multiply both sides by $(1-x)$

$$1 - x + x - x^2 + x^2 - x^3 \dots + x^n - x^{n+1} = 1 - x^{n+1}$$

This is a telescoping series so

$$1 \cancel{-x} \cancel{+x} \cancel{-x^2} \cancel{+x^2} \cancel{-x^3} + \dots \cancel{+x^n} - x^{n+1} = 1 - x^{n+1}$$

All terms of $x^1 - x^n$ cancel out leaving

$$1 - x^{n+1} = 1 - x^{n+1}$$

therefore

$$1 + x + x^2 + \dots x^n = \frac{(1-x^{n+1})}{(1-x)}$$

QED

3)

a) from 2 we have

$$\sum_{n=0}^{n} x^n = \frac{1+x^{n+1}}{1-x}$$

Set $n = \infty$

$$\lim_{n\to\infty} \sum_{n=0}^{n} x^n = \lim_{n\to\infty} \frac{1+x^{n+1}}{1-x}$$

Since $0 < x < 1$

$$\lim_{n\to\infty} (1 + x^{n+1}) = 1$$

There fore

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad 0 < x < 1$$

b) $t=0$ $A(t)=0$, $T(t)=1$

It take A 1 hr to reach T's starting point at which time T made it to $1+x(1)=1+x$ so:

$t=1$ $A(t)=1$ $T(t)=1+x$

At $t=1+x$ $A(t)=1+x$ $T(t)=1+x+x^2$

At $t=1+x+x^2$ $A(t)=1+x+x^2$ $T(t)=1+x+x^3$

Continue ad infintum

$A(t)=\sum_{n=0}^{n-1} x^n$ $\qquad$ $T(t)=\sum_{n=0}^{n} x^n$

let $n=\infty$

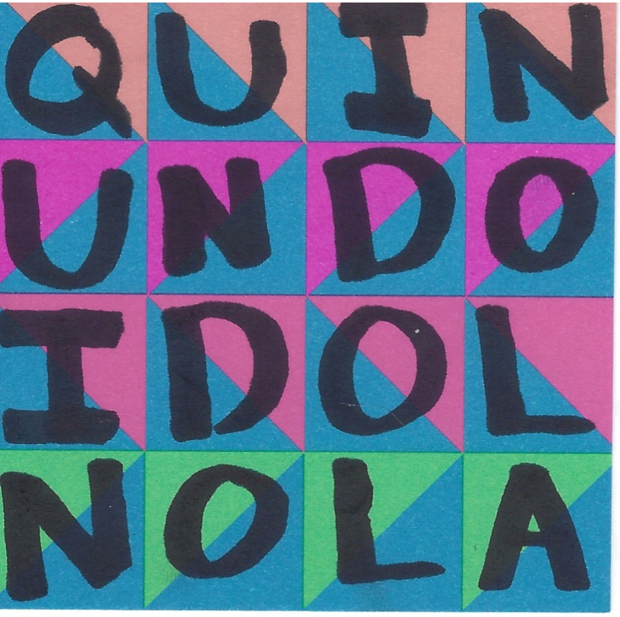
$A(t)=\lim_{n\to\infty} \sum_{n=0}^{n-1} x^n$ $\qquad$ $T(t)=\lim_{n\to\infty} \sum_{n=0}^{n} x^n = \frac{1}{1-x}$

$A(t)=t$ $\qquad$ $T(t)=1+xt$

$$t = 1+xt$$

$$t(1-x) = 1$$

$$t = \frac{1}{1-x}$$


QUIN
UNDO
IDOL
NOLA