Larry Vo
OK

HW 4

1a. At $t=0$, Distance of A is 0 while Distance of T is 50 miles.
It takes A $\frac{50}{200} = \frac{1}{4}$ hour to get to T's starting position, but by then T as gone $50 + \frac{1}{4} \cdot 50 = 62.5$ miles
So at $t = \frac{1}{4}$, D of A = 50 miles and D of T = 62.5 miles.

At time $\frac{1}{4} + \left(\frac{62.5-50}{200}\right) = \frac{5}{16}$

D of A is 62.5 while D of T is
$50 + \left(\frac{5}{16}\right)(50) = 65.625$

Next at time $\frac{1}{4} + \frac{1}{16} + \left(\frac{65.625 - 62.5}{200}\right) = \frac{21}{64}$

D of A is 65.625 while D of T is
$50 + \left(\frac{21}{64}\right)(50) = 66.40625$

And you would keep on going to see that A will never catch up to T, thus Zeno's paradox.

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1b. $200t = 50 + 50t$

$150t = 50$

$t = \frac{1}{3}$

So at $t = \frac{1}{3}$, A and T will meet together.

1C. From part a, we have

$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} \dots = \frac{1}{4}\left(1 + \frac{1}{4} + \frac{1}{16} + \dots\right)$

$= \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

Since $\left|\frac{1}{4}\right| < 1$ then

$\frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{4}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{4}\left(\frac{4}{3}\right) = \frac{1}{3}$

Which is the same answer as part b.

1D. At $t = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{21}{64}$

the Distance of A from the start is $\frac{525}{8}$ while Distance of T is $\frac{2125}{64}$

Next is $t = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{64} = \frac{11}{32}$

Now Distance of A is $\frac{525}{8} + 200\left(\frac{11}{32}\right) = \frac{1075}{8}$

And Distance of T is $\frac{2125}{64} + 50\left(\frac{11}{32}\right) = \frac{3225}{64}$

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2. $1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1-x}$

multiply by $(1-x)$

$(1-x)(1+x+x^2+ \dots + x^n) = 1-x^{n+1}$
$(1-x)+(x-x^2)+(x^2-x^3)+\dots+(x^n-x^{n+1}) = 1-x^{n+1}$
telescoping so we have
$1-x^{n+1} = 1-x^{n+1}$ which is true
thus $1+x+x^2+\dots+x^n = \frac{1-x^{n+1}}{1-x}$ is true.

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3a. For $0 < x < 1$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

From part 2, for $n \geq 0$ and any x,

$$1 + x + x^2 + \ldots + x^n = \frac{1 - x^{n+1}}{1-x}$$

and $\frac{1-x^{n+1}}{1-x} = \frac{1}{1-x}$ if x^{n+1} will go to 0. Notice that is $0 < x < 1$ then the summation will look like $1 + \frac{1}{k} + (\frac{1}{k})^2 + \ldots + (\frac{1}{k})^n$ where $x = \frac{1}{k}$, now take the limit of each of the terms as n goes to infinity and you get that all x^n go to zero thus x^{n+1} must also go to zero so if $0 < x < 1$ then $\sum_{n=0} x^n = \frac{1}{1-x}$

3b. $A = 1t$

$T = 1 + xt$ where $x < 1$

At $t = 0$, $A = 0$, $T = 1$

At $t = 1$, $A = 1$, $T = 1 + x$

At $t = 1 + x$, $A = 1 + x$, $T = 1 + (x + x^2)$

and just keep on going and this is zeno style

Now $1t = 1 + xt \rightarrow t - xt = 1$

$t(1 - x) = 1 \rightarrow t = \dfrac{1}{1-x}$

Which is when they will meet.

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4. $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n}$

To get from distance 0 to 1 you first have to travel 1/2 but from 0 to 1/2 you have to go $\frac{1}{4}$ but from 0 to $\frac{1}{4}$ you have to go $\frac{1}{8}$ and keep on going but what you are really doing is adding up all the little distances to get to 1 which is $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1$.