Homework 4.

1. $t=0$, $S(A)=0$ $S(T)=50$ miles

To get T's starting pt. $t=50/200=1/4$ hour

$t=1/4$, $S(A)=50$ $S(B)=50+25/2$

$t=1/4+1/8$ $S(A)=62.5$ $S(B)=50+12.5+6.25=68.75$?

$t=1/4+1/8+1/16$ $S(A)=68.75$ $S(B)=70.25$, A will never catch up c

(b) $200t=50t+50$

$150t=50$

$t=1/3$

A and T do meet after $1/3$ hour

(c) $1/4+1/8+\dots=1/4(1+1/2+1/4+\dots)=1/4\sum_{n=0}^{\infty}(1/2)^n$

$$\sum_{n=0}^{\infty} x^n=\frac{1}{1-x}$$

$$1/4\cdot\frac{1}{1-\frac{1}{2}}=1/2$$

(d) At the time $t=1/2+1/4+1/8+1/16+1/32=\frac{31}{32}$ hours

$S(A)=193.75$ $S(T)=194$

But the unit is $1/64$

2. $1+x+x^2+\dots+x^n=1-x^{n+1}/1-x$

$(1-x)(1+x+\dots x^n)=1-x^{n+1}$

$(1+x+\dots x^n)-(x+x^2+\dots x^{n+1})=1-x^{n+1}$

$1-x^{n+1}=1-x^{n+1}$

3. (a) $1+x+x^2+\dots+x^{n+1}=\frac{1}{1-x}$

$(1-x)(1+x+x^2\dots x^n)=1$

$(1+x+x^2\dots x^n)-(x+x^2+\dots x^n)=1$

$1=1$

(b) $t=0$

$S(A) = 0 \quad S(T) = 1$

$t=1 \quad S(A) = 1 \quad S(T) = 1+x$

$t = 1+\frac{1}{2} \quad S(A) = 1+\frac{1}{2} \quad S(T) = 1+x+\frac{1}{2x}$

$t = 1+\frac{1}{2}+\frac{1}{4} \quad S(A) = \frac{7}{4} \quad S(T) = 1+x+\frac{1}{2x}+\frac{1}{4x}$

So on, Whenever A will reach the point T was, T would go a bit further,

A will never catch up to T

2. $t = 1+xt$

$(1-x)t = 1$

$t = \frac{1}{1-x}$

A and T do meet after $\frac{1}{1-x}$ hour

4. $\sum_{n=1}^{\infty} \frac{1}{2n} = 1$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n}$

Sum(1/2^i, i=1 ... infinity)

Sum(1/n, n=1 ... ∞) = ∞

$P(n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \frac{1}{2n}$

$\lim n \to \infty, P(n) = 1$

5. $\begin{bmatrix} h & e & x & i \\ e & g & i & s \\ x & i & i & i \\ i & s & i & s \end{bmatrix}$