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1. (a) By the time that $A$ reaches the 50 mile mark where $T$ started, $T$ will still be ahead by 12 and a half miles. By the time $A$ reaches the 62 and a half mile mark, $T$ will still be ahead by another 3 and a quarter miles, etc.

   (b) $A(t) = 200t$, $T(t) = 50 + 50t$

$$\begin{aligned} 200t &= 50 + 50t \\ 150t &= 50 \\ t &= 1/3 \end{aligned}$$

   $1/3(60) = 20$, so $A$ will catch up to $T$ after 20 minutes.

   (c)

$$1/4 + 1/16 + 1/64 + 1/256 + \cdots = \frac{1/4}{1 - 1/4} = 1/3$$

   (d) If the smallest unit of time is 1/64 of an hour, T is always ahead.

| Time (hours) | Distance, A | Distance, T |
|---|---|---|
| 0 | 0 | 50 |
| 0 + 1/4 | 50 | 62.5 |
| 0+1/4+1/16 | 62.5 | 65.625 |
| 0+1/4+1/16+1/64 | 65.625 | 66.406 |

2. It is known that for any number $x$, $1 - x^{n+1} = 1 - x^{n+1}$.
For some nonnegative integer $n$, add $x - x + x^2 - x^2 + x^3 - x^3 + \cdots + x^n - x^n$, or 0, to both sides.
$1 - x^{n+1} + x - x + x^2 - x^2 + \cdots + x^n - x^n = 1 - x^{n+1} + x - x + x^2 - x^2 + \cdots + x^n - x^n$
$1 - x^{n+1} + x - x + x^2 - x^2 + x^3 - x^3 + \cdots + x^n - x^n = 1 - x^{n+1}$
The left side can be rearranged:
$1 - x + x + -x^2 + x^2 - x^3 + x^3 + \cdots - x^n + x^n - x^{n+1} = 1 - x^{n+1}$
On the left side, every 2 consecutive terms, going from left to right, are divisible by $1 - x$:

$$\begin{aligned} 1(1-x) + x(1-x) + x^2(1-x) + \cdots + x^n(1-x) &= 1 - x^{n+1} \\ (1-x)(1 + x + x^2 + \cdots + x^n) &= 1 - x^{n+1} \\ 1 + x + x^2 + \cdots + x^n &= \frac{1 - x^{n+1}}{x - 1} \end{aligned}$$

So $1 + x + x^2 + \cdots + x^n = \frac{1-x^{n+1}}{x-1}$.

3. (a) It is known that for any number $x$, $1 + x + x^2 + \cdots + x^n = \frac{1-x^{n+1}}{x-1}$. Let $x$ be between 0 and 1. Then,

$$\lim_{n\to\infty} \frac{1 - x^{n+1}}{x - 1} = \lim_{n\to\infty} \left( \frac{1}{x-1} - \frac{x^{n+1}}{x-1} \right) = \frac{1}{x-1} - \frac{0}{x-1} = \frac{1}{x-1}$$

   (b) By Zeno,

$$\begin{aligned} A(0) &= 0 & T(0) &= 1 \\ A(0 + x^0) &= 0 + x^0 & T(0 + x^0) &= x^0 + x^1 \\ A(0 + x^0 + x^1) &= 0 + x^0 + x^1 & T(0 + x^0 + x^1) &= x^0 + x^1 + x^2 \\ A(0 + x^0 + x^1 + \cdots x^n) &= 0 + x^0 + x^1 + \cdots x^n & T(0 + x^0 + x^1 + \cdots x^n) &= x^0 + x^1 + \cdots x^{n+1} \end{aligned}$$

These two distances equal each other when $x^{n+1} = 0$, which is when $n \to \infty$ since $0 < x < 1$. So $A$ and $T$ meet at time $\sum_{n=0}^{\infty} x^n$.
Algebraically, $A$ and $T$ meet the $t$ such that $A(t) = t = T(t) = 1 + xt$:

$$\begin{aligned} t &= 1 + xt \\ t - xt &= 1 \\ t(1-x) &= 1 \\ t &= \frac{1}{1-x} \end{aligned}$$

So the time when $A$ and $T$ is $\sum_{n=0}^{\infty} x^n$, or $\frac{1}{1-x}$. In other words, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

4. Before crossing some whole distance, you must cross half of it, and before crossing that you must cross half of that, etc. In other words, to cross a whole distance 1, you must cross $1/2^n + 1/2^{n-1} + 1/2^{n-2} + \cdots + 1/2^1$, where $n \to \infty$. Therefore, $1 = \sum_{n=1}^{\infty} \frac{1}{2^n}$.

5. square

| | | | | | |
|---|---|---|---|---|---|
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| a | w | h | i | l | e |
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