Homework 4

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$$\left[\begin{array}{c} 200x = 50 + 50x \\ x = 1/3 \end{array}\right]$$

1. a) At time $t = 0$, $Dist(A) = 0$
$Dist(T) = 50$
At time $t = 1/6$, $Dist(A) = 33.\overline{3}$
$Dist(T) = 58.\overline{3}$
At time $t = 1/6 + 1/12$, $Dist(A) = 50$
$Dist(T) = 62.5$
At time $t = 1/6 + 1/12 + 1/24$, $Dist(A) = 58.\overline{3}$
$Dist(T) = 64.58\overline{3}$
At time $t = 1/6 + 1/12 + 1/24 + 1/48$, $Dist(A) = 62.5$
$Dist(T) = 65.625$

A continuously approaches T without ever catching T!

b)
$$200t = 50 + 50t$$
$$150t = 50$$
$$t = 1/3$$

c)
$$\frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \ldots = \frac{1}{6}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right) = \frac{1}{6}\sum_{n=0}^{\infty} \frac{1}{2^n}$$

$$\frac{1}{6}\sum_{n=0}^{\infty} \frac{1}{2^n} = \frac{1}{6} \cdot \frac{1}{1 - 1/2} = 1/3$$

d) The above calculation (for a) goes as small as possible while staying $> 1/64$. It cannot half past $1/48$, since the next subdivision would be $1/96$. In one unit greater than $1/96$, it will catch up!

2. $(1+x+x^2+\ldots+x^n)(1-x) = \left(\frac{1-x^{n+1}}{\cancel{1-x}}\right)(\cancel{1-x})$

$(1-x)+x(1-x)+x^2(1-x)+\ldots+x^n(1-x) = 1-x^{n+1}$

$1-\cancel{x}+\cancel{x}-\cancel{x^2}+\cancel{x^2}-\cancel{x^3}+\cancel{\ldots}+x^n-x^{n+1} = 1-x^{n+1}$

$1-x^{n+1} = 1-x^{n+1}$

3. $\sum_{n=0}^{\infty} x^n = 1/1-x$

a) Assuming a typo, meaning $x = .2$,

$.2^0+.2^1+.2^2+.2^3+.2^4+.2^5\ldots = 1/1-.2$

$1+.2+.04+.008+.0016+.00032\ldots = 1.25$

$1.24992+\ldots = 1.25$

b) At time $t=0$, $Dist(A)=0$ $[t = 1+xt]$

$Dist(T)=1$

At time $t=1/2$, $Dist(A)=1/2$

$Dist(T)=1+\frac{1}{2}x$

At time $t=\frac{1}{2}+\frac{1}{4}$, $Dist(A)=3/4$

$Dist(T)=1+\frac{3}{4}x$

At time $t=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, $Dist(A)=7/8$

$Dist(T)=1+\frac{7}{8}x$

$t = 1+xt$

$t-xt = 1$

$t(1-x) = 1$

4. $\sum_{n=1}^{\infty} 1/2^n = 1$

$$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \dots + \frac{1}{2^n} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots + \frac{1}{2^n} = 1$$

$$\frac{63}{64} + \dots + \frac{1}{2^n} = 1$$

The dichotomy paradox dictates that a moving object must reach a halfway point before arriving at its destination. Here, the object continually arrives at halfway points before inevitably arriving at its' destination.

5.

D	A	N	I	E	L
A	D	E	S	T	E
N	E	L	S	O	N
I	S	S	E	I	S
E	T	O	I	L	E
L	E	N	S	E	S