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ok to post

Homework 4

1. Bike A - 200 mi/hr $A(t=0)=0$

Bike T - 50 mi/hr $\cancel{B}(t=0)=50$

a) Consider position of A and T at different times:

$t=0$ $\quad A(0)=0$

$\quad T(0)=0$

$t=0+\frac{1}{4}$ $\quad A(\frac{1}{4})=50$

$\quad T(\frac{1}{4})=50+\frac{50}{4}$

$t=0+\frac{1}{4}+\frac{1}{16}$ $\quad A(\frac{5}{16})=50+\frac{200}{16}$

$\quad T(\frac{5}{16})=50+\frac{50}{4}+\frac{50}{16}$

$\vdots$

$t'=0+\frac{1}{4^1}+\frac{1}{4^2}+\dots+\frac{1}{4^n}$ $\quad A(t')=\frac{200}{4}+\frac{200}{4^2}+\dots+\frac{200}{4^n}$

$\quad T(t')=50+\frac{50}{4}+\frac{50}{4^2}+\dots+\frac{50}{4^n}$

Therefore, bike A will always be behind T.

b) Since bike A starts at position 0 and moves at 200 mi/hr after t hours $A(t)=0+200t$.
Bike T starts at position 50 and moves at 50 mi/hr, so after t hours: $T(t)=50+50t$.

Now set $A(t)=T(t)$

$$0+200t=50+50t$$

$$200t-50t=50$$

$$150t=50$$

$$t=1/3.$$

Thus bikes A and T meet at 1/3 hour.

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c) Times used for calculation are of the form:

$$t = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^n} + \dots$$

$$= \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n.$$

Using the formula for geometrical series:

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ we get:

$$\frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{4} \cdot \frac{1}{1 - 1/4} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}.$$

d) If $1/64$ hours is the smallest unit of time, then consider: $t_1 = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{21}{64}$

and $A\left(\frac{21}{64}\right) = \frac{525}{8} = 65.625 < T\left(\frac{21}{64}\right) = \frac{2125}{32} = 66.4$

Now we can't take a unit less than $1/64$ hr so now consider $t_2 = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{64} = \frac{22}{64}$

and $A\left(\frac{22}{64}\right) = 68.75 > T\left(\frac{22}{64}\right) = 67.1875$

Therefore, bike A does pass bike T.

2. Prove that

$$1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1-x}$$

$$(1-x)(1+x+x^2+\dots+x^n) = \left(\frac{1-x^{n+1}}{1-x}\right)(1-x) \quad \text{multiply by } (1-x) \text{ both sides}$$

$$1\cdot(1+x+x^2+\dots+x^n) - x(1+x+x^2+\dots+x^n) = 1 - x^{n+1}$$

$$(1+x+x^2+\dots+x^n) - (x + x^2 + x^3 + \dots + x^{n+1}) = 1 - x^{n+1}$$

Cancel out the terms:

$$1 - x^{n+1} = 1 - x^{n+1} \quad \checkmark.$$

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3. Prove $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ if $0<x<1$

a) Proof:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots + x^n + \dots$$

Since for some infinite n,

$$\sum_{i=1}^{n} x^i = 1 + x + x^2 + \dots + x^n = \frac{1-x^{n+1}}{1-x}.$$

To find the infinite case take the limit:

$$\lim_{n\to\infty} \sum_{i=1}^{n} x^i = \lim_{n\to\infty} \frac{1-x^{n+1}}{1-x}.$$

$$\sum_{i=1}^{\infty} x^i = \frac{1 - \lim_{n\to\infty} x^{n+1}}{1-x} = \frac{1-0}{1-x}$$

since as $n\to\infty$, $x^{n+1}\to 0$ because $0<x<1$.

Therefore:

$$\sum_{i=1}^{\infty} x^i = \frac{1}{1-x}. \quad //$$

b) Zeno style: $A(0) = 0$ and $A(t) = 1t$

$T(0) = 1$ and $T(t) = 1 + xt$ where $x<1$

At time

$t=0$	$A(0)=0$	$T(0)=1$
$t=\frac{1}{x}$	$A(\frac{1}{x}) = \frac{1}{x}$	$T(\frac{1}{x}) = 1+1$
$t=\frac{1}{x}+\frac{1}{x^2}$	$A(\frac{1}{x}+\frac{1}{x^2}) = \frac{1}{x}+\frac{1}{x^2}$	$T(\frac{1}{x}+\frac{1}{x^2}) = 1+1+\frac{1}{x}$
$\vdots$		

$t' = \frac{1}{x} + \frac{1}{x^2} + \dots + \frac{1}{x^n}$ $\quad A(t') = \frac{1}{x}+\frac{1}{x^2}+\dots+\frac{1}{x^n}$

$T(t') = 1+1+\frac{1}{x}+\dots+\frac{1}{x^{n-1}}.$

Therefore, they never meet.

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4. $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$

- If Atlanta wants to get to the end of the path she must first get to $1/2$. before she does that she must get to $1/4$, even before that to $1/8$. Therefore Atlanta must pass through

$\frac{1}{2^n}$ then $\frac{1}{2^{n-1}}$, then ... $1/2$, to get to the final destination.

Thus. $\frac{1}{2^n} + \frac{1}{2^{n-1}} + \frac{1}{2^{n-2}} \cdots + \frac{1}{2} = 1.$

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Chapter III

1. Bronze Age → Age of Iron

→ 900 BC

- disappearance of Minoan and the Hittie empires
- deflation of power of ~~Greece~~ Babylonia and Egypt
- emergance of the Hebrews, Assyrians, Phoenicians and the Greeks.
- replacement of clumsy script of the Ancient Orient by easy to learn alphabet
- introduction of coined money

Age of Iron - improved trade

Age of Iron

- civilization of Greece
  → trading towns for independent politically conscious merchants
- rise of Greek polis - self governing city state
- trade connected all of the civilizations and towns
- creation of the merchant trader
  - independent, lived in a period of geographical discovery, no absolute monarchs / dieties
  - stimulate mysticism as well as rationalism and scientific discovery

2. Ionian rationalism

- mathematics addressed the 'how?' and the 'why?'
- father of Greek mathematics - Thales of Milete
  - merchant, visited Babylon + Egypt in 6th century

- Thales of Milete
    - symbol of the circumstances under which developed modern mathematics.
- math → understanding of man's purpose in life
    - order in chaos, logical chains
    - rational science (opposite of Orientalism)
        → began investigating all the why's.
- no primary sources for development of Greek math
    - classical scholarship - restauration of resources:
      Euclid, Archimedes, Apolinois... etc
        → fully developed math
        → hard to trace progress. historically.

3. 6th century B.C:
- ruins of Assyrian empire → Persia and Achaemenides
    → conquered Anatolian towns
- Greek victory ⇒ expansion + hegemony of Athens
- Perikles (5th century B.C) → democracy
- The Golden Age of Greece.
    - 'sophists' - understand math rather than just utilize it
        → teachers and philosophers
        → foundation of exact thinking
        - Hippokrates of Chios
            → writing on 'lunulae' (little moons)
            - the only complete math fragment
            - axiomatics, deduction, reasoning

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- Hippokrates ...
  - 'The Elements' ~ Greek axiomatic treatises
    - ~ Eucledian tradition (but older by a century)
  - problem of the quadrature of the circle, that is, to find the square of an area equal to that of a given square
    - ~> system of plane geometry
    - → can't be solved through finite examples
      - → new advancements - discovery of conic sections, cubic + quartic curves, quadratix
  - rationality, algebraic numbers, group theory
    - → problems presented as puzzles / anecdotes

4. Pythagoreans ~ followers of Pythagoras, a mystic, scientist and aristocratic statesman
   - → study of the unchangeable elements
     - eternal laws of universe
   - Archytas of Tarentum
     - math = high speculative science
     - numbers divided into odd, even, even-times-even ...
     - → Triangular numbers
       - link between geometry + arithmetic

         1 3 6 10 ...
   - mysticism - tried to reduce all relations to number relations
   - probably the first supplied proof of Pythagorean thm
     - result of interest in $\frac{a}{b} = \frac{b}{c}$ - can't express as 'number'
     - only knew of rational numbers (i.e. $\frac{p}{q}$)

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- 5^{th} century B.C - discovery of irrational numbers
- Zeno of Elea, a pupil of Parmenides
  - reason only recognizes absolute being and change is only apparent
  - paradoxes came in conflict with understanding of infinity
    - challenged that $\infty \times \varepsilon = \infty$ and $\infty \times 0 = 0$.
  - Paradoxes:
    1. Achilles - Achilles goes twice as fast - Tortoise starts ahead $\Rightarrow$ never meet . .
    2. Dichotomy A C D E B
       → can divide into infinitely $1/2$'s . . .
    3. Stadium
  - finite segment can be broken into infinite number of small segments of finite length
  - difficult to say line composed of points
    - against Pythagorean space as sum of points
- Discovery of irrational numbers
  - Was mathematics as an exact science possible?
  - 'A veritable logical scandal' - a crisis in Greek math
    - = Peloponnesian war $\Rightarrow$ fall of Athens (404)
      - doom of slave-owning democracy
      - introduction of aristocratic supremacy.

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