HW 4 Abdul-Ahad Butt Ok to post

1) Driver A: constant 200 mph
Driver T: constant 50 mph
T gets 50-mile head start.

a) At the beginning $t=0$,
Starting: A's distance is 0, T's distance is 50

It takes A $\frac{50}{200} = \frac{1}{4}$ to get to T's starting pt.

but at that point, T made it to $50 + \frac{1}{4} \cdot 50$
$= 62.5$

- At time $t = 1/4$ hrs:
A's distance is 50 T's distance is 62.5

- At time $t = 1/4 + \frac{1}{8} = \frac{3}{8}$ hrs.

A's distance is 62.5 T's distance is 68.75

- At time $t = 1/4 + 1/8 + 1/16 = \frac{7}{16}$ hrs

A's distance is 68.75 T's distance is 71.88
- At time $t = 1/4 + 1/8 + 1/16 + 1/32 = \frac{15}{32}$
A's distance is 71.88 T's distance is 73.44

And so on, we see that T will always still be ahead of A, based on infinitum.

b) After t hours, the distance of A is $200t$, and the distance of T from the starting point is $50+50t$.

$$\begin{aligned} 200t &= 50 + 50t \\ -50t & \quad\quad -50t \\ \hline 150t &= 50 \end{aligned}$$

$$\frac{150t}{150} = \frac{50}{150} \qquad t = 1/3$$

They will meet in $1/3$ of an hour.

c) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} \cdots = \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \rightarrow \frac{1}{4} \cdot \frac{1}{1-\frac{1}{4}} = \frac{1}{4} \cdot \frac{4}{3} = 1/3 \checkmark$$

d) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{21}{64}$ hrs

A's distance from the start is 21 T's distance from start $21\frac{1}{4}$

~~The next steps requires $\frac{1}{256}$, but it is meaningless~~

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{64} = \frac{11}{32}$$

A's distance $21 + \frac{11}{32} = 21.34$ T's distance 21.5

They do not catch up.

2) for any int $n \geq 0$ for any number x

$$1 + x + x^2 + \dots + x^n = \frac{1-x^{n+1}}{1-x}$$

$x(1-x)$ $\cdot x(1-x)$

$$\frac{x^n}{1} \cdot \frac{1-x^{n+1}}{1} x^{n+1} = 1-x^{n+1}$$

$-x^{n+1}$

3) $0 < x < 1$, the value of the infinite sum

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

a) using 2

$$\frac{1}{1-2} = \frac{1}{-1} = -1$$

b)

4) dichotomy paradox

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \dots = 1$$

we see the numbers incresingly getting smaller adding up to 1.