Homework 3

1. a) when $k=1$ $\quad 2\cdot 1-1=1^2$ is correct

when $k=n$ is correct $\quad 1+3+\cdots+2n-1=n^2$

when $k=n+1$ $\quad 1+3+\cdots+2n-1+2n+1=n^2+2n+1=(n+1)^2$

so $\sum_{k=1}^{n} 2k-1=n^2$ is correct

c) Assume $A(n)=ax^2+bx+c$

$A(1)=1=a+b+c$

$A(2)=4a+2b+c=4$

$A(3)=9a+3b+c=9$

so $a=1 \quad b=0 \quad c=0$

so $A(n)=n^2$

when $n=1$ $\quad 1=1^2$ correct.

$n=2$ $\quad 1+3=4=2^2$ correct

$n=3$ $\quad 1+3+5=9=3^2$ correct

$n=4$ $\quad 1+3+5+7=16=4^2$ correct

2. when $k=1$ $\quad 1=\frac{1(1+1)}{2}=1$ correct

when $k=n$ $\quad 1+\cdots+n=\frac{n(n+1)}{2}$ Assume this is correct

when $k=n+1$ $\quad 1+\cdots+n+n+1=\frac{n(n+1)}{2}+n+1$

$=\frac{n(n+1)+2(n+1)}{2}=\frac{(n+1)(n+2)}{2}$ is true

so $\sum_{k=1}^{n} k=\frac{n(n+1)}{2}$ is true.

b) the link is 404 cannot found

c) when $n=1$ $\quad 1=\frac{1(1+1)}{2}=1$ correct

when $n=2$ $\quad 1+2=3=\frac{2(2+1)}{2}=3$ correct

when $n=3$ $\quad 1+2+3=6=\frac{3(3+1)}{2}=6$ correct

when $n=4$ $\quad 1+2+3+4=10=\frac{4(4+1)}{2}=10$ correct.

3. $A(n) = ax^3+bx^2+cx+d$

$A(1) = a+b+c+d = 1$

$A(2) = 8a+4b+2c+d = 5$

$A(3) = 27a+9b+3c+d = 14$

$A(4) = 64a+16b+4c+d = 30$

so $a=\frac{1}{3}$ $b=\frac{1}{2}$ $c=\frac{1}{6}$ $d=0$

so $A(n) = \frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n$

a) when $k=1$ $A(1)=\frac{1}{3}+\frac{1}{2}+\frac{1}{6}=1=1^2$ correct

Assume $k=n$ is correct $1+4+\cdots+n^2 = \frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n = \frac{(2n+1)(n+1)n}{6}$

when $k=n+1$ $1+\cdots+n^2+(n+1)^2 = \frac{(2n+1)(n+1)n}{6}+(n+1)^2$

$= \frac{(n+1)(2n^2+n)+6(n+1)(n+1)}{6}$

$= \frac{(n+1)(2n^2+n+6n+6)}{6} \quad \frac{(n+1)(2n^2+7n+6)}{6}$

$= \frac{(n+1)(2n+3)(n+2)}{6}$

so it is correct

b) when $n=1$ $1^2 = \frac{1\cdot2\cdot3}{6}=1$ correct

when $n=2$ $1+2^2=5=\frac{2\cdot3\cdot5}{6}=5$ correct

$n=3$ $1+2^2+3^2=14=\frac{3\cdot4\cdot7}{6}=14$ correct

$n=4$ $1+2^2+3^2+4^2=30=\frac{4\cdot5\cdot9}{6}=30$ correct

4. when $k=1$ $1=(\frac{1\cdot2}{2})^2=1$ correct

Assume $k=n$ is correct $1+\cdots+n^3=(\frac{n(n+1)}{2})^2$

when $k=n+1$ $1+\cdots+n^2+(n+1)^2 = (\frac{n(n+1)}{2})^2+(n+1)^2$

$= \frac{n^2(n+1)^2}{4}+\frac{4(n+1)^2}{4} = \frac{(n+1)^2(n^2+4)}{4}$