Vivian Choong
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Homework 3

(1) (a) $\sum_{k=1}^{1} 2k-1 = 1^2$

Base case: $n=1$

$2(1)-1 = 1^2$

$1 = 1$ ✓

Inductive Case:

$\sum_{k=1}^{m+1} 2k-1 = m^2 + 2(m+1)-1$

$\sum_{k=1}^{m+1} 2k-1 = m^2 + 2m+1$

$\sum_{k=1}^{m+1} 2k-1 = (m+1)^2$

(b)
$$\begin{matrix} x & x & x \\ x & x & x \\ x & x & x \end{matrix}$$
Each of the colored portions are composed of $2n-1$ crosses. We can generalize the summation of those x's to n^2

(c) $S(1) = \sum_{k=1}^{1} 2k-1 = 1^2$

$2(1)-1 = 1^2$

$1=1$ ✓

$S(2)$: $\sum^{2} 2k-1 = 2^2$

$2(1)-1+2(2)-1 = 4$

$2-1+4-1 = 4$

$4=4$ ✓

$S(3)$: $\sum 2k-1 = 3^2$

$2(1)-1+2(2)-1+2(3)-1 = 9$

$9=9$ ✓

(2) (a) $\frac{n(n+1)}{2}$

Base case - $S(1)$

$\frac{1(1+1)}{2} = 1$

$1=1$ ✓

Inductive Case:

$\sum_{k=1}^{m} = \frac{m(m+1)}{2}$

$\sum_{k=1}^{m+1} = \frac{m^2+m}{2} + \frac{2m+2}{2}$

$\sum_{k=1}^{m+1} = \frac{m^2+3m+2}{2}$

(b) $\sum_{k=1}^{n} = 1+2+3+\dots$

$\frac{(n-1)(n+1)}{2} + \frac{n+1}{2} = \frac{(n-1)(n+1)+(n+1)}{2}$

$\sum_{k=1}^{n} k = \frac{(n+1)(n)}{2}$

(c) $S(1)$ $\sum_{k=1}^{1} k = \frac{1(1+1)}{2}$

$1 = 2/2$

$1=1$ ✓

$S(2)$ $\sum_{k=1}^{2} k = \frac{2(2+1)}{2}$

$3 = 6/2$

$3=3$ ✓

$S(3) = \sum_{k=1}^{3} k = \frac{3(3+1)}{2}$

$6 = 12/2$

$6=6$ ✓

(3) $\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} = \frac{2n^3 + 3n^2 + n}{6}$

(a) Proof By induction

Base Case: $n=1$

$$1^2 = \frac{2(1)^2 + 3(1)^2 + 1}{6}$$

$1 = 1$ ✓

Inductive case:

$$\sum_{k=1}^{m} k^2 = \frac{2m^3 + 3m^2 + m}{6}$$

$$\sum_{k=1}^{m+1} k^2 + (m+1)^2 = \frac{2m^3 + 3m^2 + m}{6} + (m+1)^2$$

$$\sum_{k=1}^{m+1} k^2 = \frac{2(m+1)^2 + 3(m+1)^2 + (m+1)}{6}$$

(b) $S(1) = 1^2 = \frac{2+3+1}{6}$

$1 = 1$ ✓

$S(2) \to 2^2 + 1 = \frac{2(2^3) + 3(2^2) + 2}{6}$

$5 = 5$ ✓

$S(3) \to 3^2 + 2^2 + 1 = \frac{2(27) + 3(9) + 3}{6}$

$14 = 14$ ✓

$S(4) = 4^2 + 3^2 + 2^2 + 1 = \frac{2(64) + 3(16) + 4}{6}$

$30 = 30$ ✓

(4) $\sum k^3 = \left(\frac{n+(n+1)}{2}\right)^2$

Base case: $n = 1$

$$1^3 = \left(\frac{(1)(2)}{2}\right)^2$$

$1 = 1$ ✓

Inductive case:

$$1^3 + 2^3 + \cdots + m^3 = \left(\frac{(m)(m+1)}{2}\right)^2$$

$$1^3 + 2^3 + \cdots m^3 + (m+1)^3 = \left(\frac{m(m+1)}{2}\right)^2 + (m+1)^3$$

$$1^3 + 2^3 + \cdots + m^3 + (m+1)^3 = \left(\frac{(m+1)(m+2)}{2}\right)^2$$