Sarah Catovic Homework 3 9/19/2021

Summary of Chapter 2 Sections 6-7: When we look at mathematics now, there's a lot of focus on procedural understanding rather than trying to understand how it's different theorems and formulas came about. The oldest indian texts that are still around are from early AD. Indians used a decimal system without a place value which was formed by the Brahmi numerals, which go back to King Asoka (300 BC). Sulvasutras, which go back too 500 BC which have different mathematical rules, mainly with geometry. They deal with the construction of altars, but include how to make rectangles and ways to know the relationships between the diagonal and sides.

Some of their approximations:

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\cdot 4} - \frac{1}{3\cdot 4\cdot 34} (= 1.4142156)$$

$$\pi = 4\left(1 - \frac{1}{8} + \frac{1}{8\cdot 29} - \frac{1}{8\cdot 29\cdot 6} + \frac{1}{8\cdot 29\cdot 6\cdot 8}\right)^2 = 18(3 - 2\sqrt{2})$$

These don't occur in later Hindu works shows it may not be continuous. Janism encouraged mathematical study, as can be seen by showing $\pi = \sqrt{10}$ in some of their sacred books.

1. a) $\sum_{k=1}^{n} 2k-1 = n^2$ Base case: $n=1$ $2(1)-1 = 1 = 1^2$ Yes!

Assume that if it works for $k \geq 1$ then it must hold for $k+1$

$1 + 2 + \cdots + 2k-1 = k^2 \rightarrow$ Assumed to be true

$\underline{1 + 2 + \cdots + 2k-1} + 2k+1 = (k+1)^2$

$k^2 + 2k+1 = (k+1)^2$ which is true so it holds for $k+1$

b)

c) Since it is of degree 2, we need to show that they agree at 3 different values

$n=1$ $2(1)-1 = 1^2$ ✓

$n=2$ $2(1)-1 + 2(2)-1 = 4 = 2^2$ ✓

$n=3$ $1+3+ 2(3)-1 = 9 = 3^2$ ✓

2.a) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ Base case $n=1$ $1 = \frac{1(2)}{2} = 1$ Yes!

If we assume it is true for all k $k \geq 1$, then it must be true for $k+1$

So $1+2+\dots+k = \frac{n(n+1)}{2}$ is assumed to be true

$1+2+\dots+k+k+1 = \frac{(k+1)(k+2)}{2}$

$\frac{k(k+1)}{2} + k+1 = \frac{(k+1)(k+2)}{2}$

$\frac{k(k+1)+2k+2}{2} = \frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$ which is true

so it holds for $k+1$

b) $1 + 2 + \dots \frac{n}{2}$ $\frac{n}{2}$ copies of $n+1$ (since each

$n + n-1 + \dots \frac{n}{2}$ of the columns add up to $n+1$ so

the total is is $(\frac{n}{2})(n+1)$)

c) Since it is of degree 2, we just need to show that it agrees at 3 different values

$n=0$ $0 = 0(\frac{1}{2})$ ✓

$n=1$ $0+1 = \frac{1(2)}{2}$ ✓

$n=2$ $0+1+2 = \frac{2(3)}{2} = 3$ ✓

3.a) $\sum_{k=1}^{n} k^2 = A(n)$ Since this is of degree 2, $A(n)$ must be of degree 3 so in the form $A(n) = an^3 + bn^2 + cn + d$

$A(0) = 0$, $A(1) = 1$, $A(2) = 5$, $A(3) = 15$

$A(0) = a(0)^3 + b(0^2) + c(0) + d = d$ $d = 0$

$A(1) = a(1^3) + b(1^2) + c(1) + d = 1$ $a+b+c = 1$

$A(2) = a(2^3) + b(2^2) + c(2) + d = 4$ $8a + 4b + 2c = 5$

$A(3) = a(3^3) + b(3^2) + c(3) + d = 15$ $27a + 9b + 3c = 15$

$$\begin{array}{r} 8a+4b+2c=5 \\ -\ 2a+2b+2c=2 \\ \hline 6a+2b=3 \end{array} \qquad \begin{array}{r} 27a+9b+3c=15 \\ -3a-3b-3c=15 \\ \hline 24a+6b=0 \end{array} \qquad \begin{array}{r} 24a+6B=0 \\ -\ 24a+8b=0 \\ \hline -2b=0 \\ b=0 \end{array}$$

$a = \frac{1}{2}$ $c = \frac{1}{2}$ $b = 0$ $d = 0$ so $A(n) = \frac{1}{2}n^3 + \frac{1}{2}n = \frac{1}{2}n(n^2+1)$

b) $\sum_{k=1}^{n} k^2 = \frac{n(n^2+1)}{2}$ Base case $n=1$ $1^2 = \frac{1(2)}{2}$ Yes!

If we assume that it is true for all $k \geq 1$, then it must be true for $k+1$

$1+2+\cdots+k^2 = \frac{k(k^2+1)}{2}$ is assumed to be true.

$1+2+\cdots+k^2+(k+1)^2 = \frac{(k+1)((k+1)^2+1)}{2}$

$\frac{k(k^2+1)}{2} + (k+1)^2 = \frac{(k+1)(k^2+2k+2)}{2} = \frac{k^3+2k^2+2k+k^2+2k+2}{2}$

$\frac{k(k^2+1)+2(k+1)^2}{2} = \frac{k^3+k+2k^2+4k+2}{2} \neq$

so it is true for all k

c) Since this of degree 3, we just need to show that it agrees for 4 different values:

$n=1$ $1^2 = \frac{1(2)}{2} = 1$ ✓ $n=3$ $1^2+2^2+3^2 = \frac{3(10)}{2}$

$n=2$ $1^2+2^2 = \frac{2(5)}{2}$ ✓ $n=4$ $1^2+2^2+3^2+4^2 = \frac{4(17)}{2}$

4. $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$ Base case: $n=1$ $1^3 = \left(\frac{1(2)}{2}\right)^2$ Yes!

If we assume it is true for $k \geq 1$, then it must be true for $k+1$

$1+2+\cdots+k^3 = \left(\frac{k(k+1)}{2}\right)^2 \rightarrow$ assume this to be true

$1+2+\cdots+k^3+(k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$\frac{(k^2+k)^2}{4} + (k+1)^3 = \frac{k^4+2k^3+k^2}{4} + 4(k+1)(k^2+2k+1)$

$\frac{(k^2+k)^2+4(k+1)^3}{4} = \frac{(k+1)^2(k^2+4k+4)}{2}$

$= \left(\frac{(k+1)(k+2)}{2}\right)^2$ So it is true for all k