Homework III

(1) $\sum_{k=1}^{n} 2k-1 = n^2$

(a) <u>induction:</u>

base case $n=1$: $2(1)-1 = 1^2$

$1 = 1$ ✓

inductive case:

assuming it is true for $n-1$

$(n-1)^2 + (2n-1) = n^2$

$n^2 - 2n + 1 + 2n - 1 = n^2$

$n^2 = n^2$ ✓

(b) <u>geometry:</u>

$1+3+5+7+9 = 25$ (or 5^2)

$1+3+5+7 = 16$ (or 4^2)

$1+3+5 = 9$ (or 3^2)

$1+3 = 4$ (or 2^2)

$1 = 1$ (or 1^2)

(c) <u>Dr Z's way:</u>

$p(k)$ $2k-1$ is a polynomial of degree 1

so $\sum_{i=1}^{n} p(i)$ is a polynomial of degree 2

$n=1$: $1 = 1$

$n=2$: $1+3=4$

$n=3$: $1+3+5=9$

$n=4$: $1+3+5+7=16$

$n=5$: $1+3+5+7+9=25$

n: n^2 as shown above

② $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

(a) induction:

base case $n=1$:

$1 = \frac{1(1+1)}{2} = 1$ ✓

Assuming it is true for $n-1$ we show it for n

$\frac{(n-1)(n-1+1)}{2} + n = \frac{n(n+1)}{2}$

$\frac{(n-1)n}{2} + n = \frac{n(n+1)}{2}$

$\frac{(n-1)n + 2n}{2} = \frac{n(n+1)}{2}$

$\frac{n^2 - n + 2n}{2} = \frac{n^2+n}{2}$

$\frac{n^2+n}{2} = \frac{n^2+n}{2}$ ✓

(b) Gauss:

$1+2+\dots+100 = 5050$

$1+2+3+\dots+50$

$100+99+98+\dots+51$

$5050 = 50(101) = \frac{100(101)}{2}$

(c) Dr. Z:

$n=1$: $1 = \frac{1(2)}{2}$

$n=2$: $1+2 = 3 = \frac{2(3)}{2}$

$n=3$: $1+2+3 = 6 = \frac{3(4)}{2}$

$n=4$: $1+2+3+4 = 10 = \frac{4(5)}{2}$

$\vdots$

n: $\frac{n(n+1)}{2}$

③ $\sum_{k=1}^{n} k^2$

$n=1: \; 1^2 = 1$

$n=2: \; 1^2+2^2 = 5$

$n=3: \; 1^2+2^2+3^2 = 14$

$n=4: \; 1^2+2^2+3^2+4^2 = 30$

$n=5: \; 1^2+2^2+3^2+4^2+5^2 = 55$

$n=6: \; 1^2+2^2+3^2+4^2+5^2+6^2 = 91$

$n: \; \dfrac{n(n+1)(2n+1)}{6}$

(a) induction:

base case $n=1$:

$$1 = \frac{1(1+1)(2(1)+1)}{6} = 1 \checkmark$$

assuming it is true for $n-1$, we show for n

$$\frac{(n-1)(n-1+1)(2(n-1)+1)}{6} + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(2n+1)}{6} \checkmark$$

(b) DIZ:

$n=1 = \dfrac{1(2)(3)}{6} = 1$

$n=2 = \dfrac{2(3)(5)}{6} = 5$

$n=3 = \dfrac{(3)(4)(7)}{6} = 14$

$n=4 = \dfrac{(4)(5)(9)}{6} = 30 \checkmark$

$n: \dfrac{n(n+1)(2n+1)}{6}$

(4) $$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

base case $n=1$:

$$1^3 = \left(\frac{1(1+1)}{2}\right)^2 = 1 \checkmark$$

assuming it is true for $n-1$, we shown:

$$\left(\frac{(n-1)(n-1+1)}{2}\right)^2 + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\left(\frac{(n-1)(n)}{2}\right)^2 + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\left(\frac{n^2-n}{2}\right)^2 + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\frac{n^4-2n+n^2}{4} + n^3 = \left(\frac{n^2+n}{2}\right)^2$$

$$\frac{n^4-2n+n^2}{4} + n^3 = \frac{n^4+2n+n^2}{4}$$

$$\frac{n^4-2n+n^2+4n^3}{4} = \frac{n^4+2n+n^2}{4}$$

$$\frac{n(n^3+4n^2+n-2)}{4} = \frac{n(n^3+n+2)}{4} \quad \ldots = \left(\frac{n(n+1)}{2}\right)^2$$