H.W 3

1. $\sum_{k=1}^{n} 2k-1=n^2$

a. base step

$\sum_{k=1}^{1} 2k-1=1$, $1^2=1$

Now Assume that $\sum_{k=1}^{n} 2k-1=n^2$ for some

$n\in\mathbb{N}$. Lets prove that $\sum_{k=1}^{n+1} 2k-1=(n+1)^2$.

First $\sum_{k=1}^{n+1} 2k-1=\sum_{k=1}^{n} 2k-1+2(n+1)-1=n^2+2(n+1)-1$

by our assumption. Now $n^2+2(n+1)-1=n^2+2n+2-1$
$=n^2+2n+1=(n+1)^2$ which we wanted to
prove. Thus $\sum_{k=1}^{n} 2k-1=n^2$ for all $n\in\mathbb{N}$.

b. Start of with $m=1$ as ◇ = □

$m=2^2=4$ as

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$m=3^2=9$ as

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□ □ □ = □ □ ◇
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where I am keeping the squares of the last number and changing the remaining to diamonds so

$m=4^2=16$ as

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We notice that n^2 = all of the diamonds at n^2 and before so for 4^2, there are 7 diamonds plus 5+3+1 so there is a total of 16 diamonds. This is how to do it with geometry.

c. n^2 is degree 2 so
$S(n) = \sum_{k=1}^{n} 2k-1$ is a polynomial
of degree $k+1 = 3$.
So $1^2 = 1$, $2^2 = 4$, $3^2 = 9$
and $\sum_{k=1}^{1} 2k-1 = 1$, $\sum_{k=1}^{2} 2k-1 = 4$, $\sum_{k=1}^{3} 2k-1 = 9$

So since both are degree 2 and
they have the same values for 3 different
n, then $\sum_{k=1}^{n} 2k-1 = n^2$

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2a. $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

Base step: $n=1$, $\sum_{k=1}^{1} k = 1$, $\frac{(1)(1+1)}{2} = 2$

Now Assume $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ for some $n \in \mathbb{N}$.

Then prove $\sum_{k=1}^{n+1} k = \frac{(n+1)((n+1)+1)}{2}$

Simplifying RHS, $\frac{(n+1)(n+2)}{2}$

Notice that $\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + n+1$ but by our assumption, $\sum_{k=1}^{n} k + n+1 = \frac{n(n+1)}{2} + n+1$

$= \frac{n(n+1)+2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$ which we wanted to prove. Thus $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

for all $n \in \mathbb{N}$.

b. $S = 1 + 2 + 3 + \ldots + n$

$S = n + (n-1) + (n-2) + \ldots + 1$

Now add both together,

$$2S = \underbrace{(n+1) + (n+1) + (n+1) + \ldots + (n+1)}_{n \text{ terms}}$$

$2S = (n+1)(n)$

$$S = \frac{(n+1)(n)}{2} = \sum_{n=1}^{n} K$$

c. $\frac{n(n+1)}{2}$ is degree two then

$\sum_{K=1}^{n} K$ is degree $K+1$. Check up to $n=3$,

$$\frac{1(1+1)}{2} = 1, \quad \frac{2(2+1)}{2} = 3, \quad \frac{3(3+1)}{2} = 6$$

$$\sum_{K=1}^{1} K = 1, \quad \sum_{K=1}^{2} K = 3, \quad \sum_{K=1}^{3} K = 6$$

Since both are degre 2 and it has same values for up to $n=3$, then

$$\sum_{K=1}^{n} K = \frac{n(n+1)}{2}$$

3. $\sum_{K=1}^{n} K^2$

$A(n) = an^3 + bn^2 + cn + d$

$A(0) = 0$, $A(1) = 1^2 = 1$, $A(2) = 1 + 2^2 = 5$

$A(3) = 1 + 4 + 3^2 = 14$

$A(0) = a(0)^3 + b(0)^2 + c(0) + d = d$

$A(1) = a(1)^3 + b(1)^2 + c(1) + d = a + b + c + d$

$A(2) = a(2)^3 + b(2)^2 + c(2) + d = 8a + 4b + 2c + d$

$A(3) = a(3)^3 + b(3)^2 + c(2) + d = 27a + 9b + 3c + d$

$d = 0$, $a + b + c = 1$, $8a + 4b + 2c = 5$, $27a + 9b + 3c = 14$

$a = \frac{1}{3}$, $b = \frac{1}{2}$, $c = \frac{1}{6}$

$A(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + 0 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$

$= n \cdot \frac{1}{6}(2n^2 + 3n + 1) = \frac{n(2n+1)(n+1)}{6}$

a). Base step $\sum_{K=1}^{1} K^2 = 1$, $\frac{1(2(1)+1)(1+1)}{6} = \frac{6}{6} = 1$

Now assume $\sum_{K=1}^{n} K^2 = \frac{n(2n+1)(n+1)}{6}$ for some $n \in \mathbb{N}$.

Then prove $\sum_{K=1}^{n+1} K^2 = \frac{(n+1)(2(n+1)+1)((n+1)+1)}{6}$

$= \frac{(n+1)(2n+3)(n+2)}{6}$

First $\sum_{K=1}^{n+1} K^2 = \sum_{K=1}^{n} K^2 + (n+1)^2 = \frac{n(2n+1)(n+1)}{6} + (n+1)^2$

by our assumption, Now that is equal to

$\frac{n(2n+1)(n+1) + 6(n+1)^2}{6} = \frac{n(2n+1)(n+1) + 6n^2 + 12n + 6}{6}$

$= \frac{2n^3 + 3n^2 + n + 6n^2 + 12n + 6}{6} = \frac{2n^3 + 9n^2 + 13n + 6}{6}$

$$=\frac{(n+1)(2n+3)(n+2)}{6}$$

Which is what we wanted to prove.

b. $\sum_{k=1}^{n} k^2 = \frac{n(2n+1)(n+1)}{6}$

degree three so need to check up to $n=4$

$\sum_{k=1}^{1} k^2 = 1$, $\sum_{k=1}^{2} k^2 = 5$, $\sum_{k=1}^{3} k^2 = 14$

$\sum_{k=1}^{4} k^2 = 30$

$$\frac{1(2(1)+1)(1+1)}{6} = 1$$

$n=2$, 5

$n=3$, 14

$n=4$, 30

They all agree up to $n=4$ so $\sum_{k=1}^{n} k^2 = \frac{n(2n+1)(n+1)}{6}$ is true

4. $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

Degree 4 so check up to $n=5$.

$\sum_{k=1}^{1} k^3 = 1$, $\sum_{1}^{2} k^3 = 1+8 = 9$, $\sum_{1}^{3} k^3 = 1+8+27 = 36$

$\sum_{1}^{4} k^3 = 1+8+27+64 = 100$

$\sum_{1}^{5} k^3 = 1+8+27+64+125 = 225$

$n=1$, $\left(\frac{1(1+1)}{2}\right)^2 = 1$

$n=2$, 9
$n=3$, 36
$n=4$, 100
$n=5$, 225

Both degree 4 and have same values for $k+1$, thus

$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

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