5,2 W
4,4 L
5,6 L
7,2 W
7,4 W
7,6 W

9/19/21

<u>HW 3 - 437</u>

(1.) $\sum_{k=1}^{n} 2k-1=n^2$

(a.) Using mathematical induction;
<u>Base case $n=1$</u> $2(1)-1=(1)^2$, $1=1$ ✓

$S=1+3+5+\dots(2n-1)$

$S=(2n-1)+(2n-3)+\dots \Rightarrow 2S=2n+2n+\dots 2n$, $2S=(2n)(n)=\boxed{n^2}$

(b.) $S_e=2+4+6+\dots 2n$
$=2(1+2+3+n)=\frac{2(1+n)}{2}=n(1+n)$

$S_o+S_e=1+2+3+\dots 2n$

$S_o+S_e=2n+(2n-1)+\dots 1 \Rightarrow 2(S_o+S_e)=(2n+1)+(2n+1)\dots(2n+1)(2n)$

$2(S_o+S_e)=(2n+1)n$

$S_o=(2n+1)n-S_e=(2n+1)n-n(1+n)=n(2n+1)-(1+n)=n\cdot n=\boxed{n^2}$

(c) $\sum_{k=1}^{n}(2k-1)=n^2$, $\sum_{k=1}^{n}(2k-1)=\sum_{k=1}^{n}2k-\sum_{k=1}^{n}1=2\sum_{k=1}^{n}k-n$

$=2\left(\frac{n(1+n)}{2}\right)-n \Rightarrow n(1+n-1)=\boxed{n^2}$

9/19/21

HW 3-437

(2.) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

(a) Mathematical Induction

Base Case: $n=1$, $1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$ ✓

Let $P(k) = 1+2+3+ \cdots k = \frac{k(k+1)}{2}$ be true ✓

$P(k+1) = 1+2+3+ \cdots (k+1) = \frac{(k+1)((k+1)+1)}{2} \rightarrow 1+2+3+ \cdots k + (k+1) = \frac{(k+1)(k+2)}{2}$

↳

$1+2+3+ k = \frac{k(k+1)}{2} + (k+1) \rightarrow 1+2+3+ k + (k+1) = \frac{k(k+1)}{2} + (k+1)$

$= 1+2+3+ \cdots k + (k+1) = \frac{(k+1)(k+2)}{2}$, So we have proven it by mathematical induction

(b) If $S_n = 1^2 + 2^2 + \cdots n^2 \Rightarrow S_{2n} - 2S_n = ((2n)^2 - 1^2) + ((2n-1)^2 + 2^2) + \cdots ((n+1)^2 - n^2)$

$= (2n+1)(2n-1+2n-3+\cdots 1) = (2n+1)n^2$

Using Gauss's trick, $S_{2n+1} - 2S_n = (2n+1)(n+1)^2$

$S_0 = 0^2 = 0$

$S_1 = 1 + 2S_0 = 1$

$S_2 = 3 + 2S_1 = 5$

$S_4 = 2S + 2S_2 = 30$

(3.) $\sum_{k=1}^{n} k^2$