Daniel Rogers Due 9/19

Homework 3

1. $\sum_{k=1}^{n} 2k - 1 = n^2$

a) Base case : $n = 1$: $\sum_{k=1}^{1} 2(k) - 1 = 1^2$; $1 = 1$

$P(n-1)$ implies $p(n)$ | $P(1) \rightarrow P(2) \ldots$

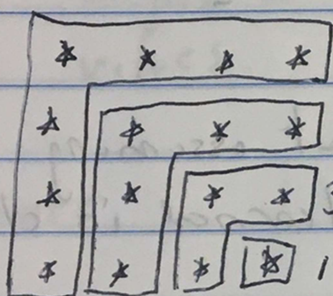
$P(n-1) = (2(1) - 1) + (2(2) - 1) + \ldots + (2(n-1) - 1) = (n-1)^2$

$(2(1) - 1) + (2(2) - 1) + \ldots + (2(n-1) - 1) = n^2 - \underbrace{2n + 1}$

$(2(1) - 1) + (2(2) - 1) + \ldots + (2(n-1) - 1) + (2n - 1) = n^2$

TRUE!

b)



$7 = 2(4) - 1$

$5 = 2(3) - 1$

$3 = 2(2) - 1$

$1 = 2(1) - 1$

c)

$n = 1$: $2(1) - 1 = 1^2$ TRUE

$n = 2$: $2(1) - 1 + 2(2) - 1 = 2^2$ TRUE

~~QED, since~~

$n = 3$: $2(1) - 1 + 2(2) - 1 + 2(3) - 1 = 3^2$ TRUE

QED, since polynomials are of degree 2 and they agree at 3 different values.

2. $\sum_{k=1}^{n} k = n(n+1)/2$

a) Base Case: $n=1$: $\sum_{k=1}^{1} k = 1(1+1)/2$

$1 = 2/2$

$1 = 1$

$P(n-1)$ implies $p(n)$

$P(n-1) = \sum_{k=1}^{n-1} k = (n-1)(n-1+1)/2$

$1+2+\dots+n-1 = n(n-1)/2$

$1+2+\dots+n-1+n = n(n-1)/2 + n$

$1+2+\dots+n-1+n = n(n+1)/2$

TRUE!

b) Link didn't work, but assuming it's the $\sum_{i=1}^{100} i$ trick we learned in class...

$1 + 2 + 3 + \dots$

$n + n-1 + n-2 + \dots$

$(= n+1)$

$\to n/2$ rectangles

$(n+1) \cdot n/2 = n(n+1)/2$

$\therefore \sum_{k=1}^{n} k = n(n+1)/2$

c) $n=1$: $1 = 1(1+1)/2$ TRUE

$n=2$: $1+2 = 2(2+1)/2$ TRUE

$n=3$: $1+2+3 = 3(3+1)/2$ TRUE

QED, since polynomials are of degree 2 and they agree at 3 different values

3. $\Sigma_{k=1}^{n} k^2$

$n=1 : 1^2 = 1$

$n=2 : 1^2+2^2 = 5$

$n=3 : 1^2+2^2+3^2 = 14$

$\Sigma_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$

$\frac{1(1+1)(2\cdot1+1)}{6} = 1$

$\frac{2(2+1)(2\cdot2+1)}{6} = 5$

$\frac{3(3+1)(2\cdot3+1)}{6} = 14$

To prove Dr. Z's way,

$n=4 : 1^2+2^2+3^2+4^2 = \frac{4(4+1)(2\cdot4+1)}{6}$ TRUE

QED, since polynomials are out of degree 3 and agree at 4 different values.

4. unsure.