Homework for Lecture 3

1. Base case $n=1$

$2-1=1$ Proved

Assuming it is true for $n-1$ then this is true for n

$P(n-1)$ implies $P(n)$

$P(n-1) = (2-1)+(3-1)+\cdots(2(n-1)-1) = (n-1)^2$

$= 1+3+\cdots+2n-3 = (n-1)^2$

$P(n)$ LHS $= 1+2+\cdots+2n-1$

$P(n)-P(n-1)$ LHS $= 2n-1$

$P(n)-P(n-1)$ RHS $= n^2-(n-1)^2 = n^2-(n^2+1-2n) = 2n-1$

LHS = RHS, $P(n-1)$ is already true. Thus, proved

(b) Sum $(i^2, i=1\cdots n) = \dfrac{n(n+1)(2n+1)}{6}$

If $1+\cdots 2k-1 = n(n+1)(2n+1)/6 \ni n^2$

$n=1 \quad 1 = 1+2+3/6 = 1$

$n=2 \quad 1+1+3 = 2\cdot3\cdot5 = 5$

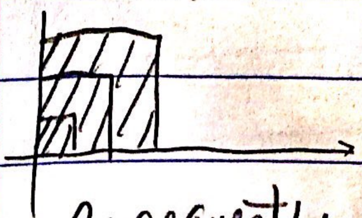
$n=3 = 1+1+3+5 = 3\cdot4\cdot5/6 = 10$

Proved

(c) This is geometry

$2k-1$ is the k^{th} odd number

n^2 is the n^{th} square number



1, 3, 5, 7, 9 could be draw like that etc.

Consequently, its area is n^2

Hence $\sum_{k=1}^{n} 2k-1 = n^2$

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2. (a) base step

$n=1, \quad 1 = 1\cdot 2/2 = 1$ Proved

Assuming it is true for n-1 then it is true for n

$P(n-1)$ implies $P(n)$

$P(n-1) = 1+2+\ldots+n-1 = n(n-1)/2$

$P(n) - P(n-1)$ LHS $= n$

$P(n) - P(n-1)$ RHS $= n(n+1)/2 - n(n-1)/2 = n$

LHS = RHS, proved

(b) If n is even, then $\sum_{k=1}^{n} k$ could be write as

$1 + 2 + \cdots + n/2$

$n \quad n-1 \cdots n/2$

add the top and bottom together we get

$n/2$ copies of $(n+1)$

Thus $\sum_{k=1}^{n} k = \frac{n}{2}\cdot(n+n+1)$

(c) $\text{Sum}(i^2, i=1,2,\cdots n) = n(n+1)(2n+1)/6$

$\text{Sum}\, i = \sqrt{\frac{n(n+1)(2n+1)}{6}}$

Thus if $\frac{n(n+1)}{2} = \frac{\sqrt{n(n+1)(2n+1)}}{6}$, we could prove

for $n=1 \quad 1 = 1$

$n=2 \quad 5=5$

Proved

3. (a)

(b) If $p(n)$ is a polynomial degree k then
$S(n) = p(1) + p(2) + \dots + p(n)$
$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$
$n=1 \quad 1=1 \qquad 1+3+\dots+(2n-1) = n^2$
$n=2 \quad 5=5$

4. base step $n=1$
$1 = (1(1+1)/2)^2 = 1$
Assuming it is true for $n-1$ then it is true for n
$P(n-1)$ implies $P(n)$
$P(n-1)$ LHS $= 1^3 + 2^3 + 3^3 \cdots (n-1)^3$
$P(n-1)$ RHS $= \left(\frac{(n-1)n}{2}\right)^2$
$P(n) - P(n-1)$ LHS $= n^3$
$P(n) - P(n-1)$ RHS $= \left(\frac{n(n+1)}{2}\right)^2 - \left(\frac{n(n-1)}{2}\right)^2$
$$= \frac{n^4 + n^2 + 2n^3 - n^4 - n^2 + 2n^3}{4}$$
$$= n^3$$

LHS = RHS, Proved