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Dr. Z, History of Math
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Homework for Lecture 2 - OK to post

① Each row and column in an $n \times n$ magic square add up to the same amount, which is given by the formula:

$$\frac{n(n^2+1)}{2}$$

Proof: Since there are n rows with the same sum, the sum of any row will be $\frac{1}{n} \cdot$ (the sum of all of the numbers in the magic square).

We use the formula $1+2+\cdots+n=\frac{n(n+1)}{2}$, but since we are adding up to n^2, since that is the largest number in the magic square, we write

$$1+2+\cdots+n^2=\frac{n^2(n^2+1)}{2}$$

This is the sum of all the numbers in the magic square, so we obtain

$$\frac{1}{n}\cdot\frac{n^2(n^2+1)}{2}=\frac{n(n^2+1)}{2}$$

which is the sum of the numbers in any row or column of the magic square.

(2) Step 1: Draw blank 3x3 grid

___	___	___
___	___	___
___	___	___

Step 2: Scaffold

		___		
	___	___	___	
___	___	___	___	___
	___	___	___	
		___		

Step 3: Write numbers on diagonals

		1		
	2	___	4	
3	___	5	___	7
	6	___	8	
		9		

Step 4: Reflect

		1		
	2	9	4	
3	7	5	3	7
	6	1	8	
		9		

2	9	4
7	5	3
6	1	8

(3) Step 1:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Step 2: Kick out the middle two numbers on the top, sides, and bottom

1			4
	6	7	
	10	11	
13			16

So we have 2, 3, 5, 8, 9, 12, 14, 15

Step 3: Use the missing numbers to fill in the spots in reverse order

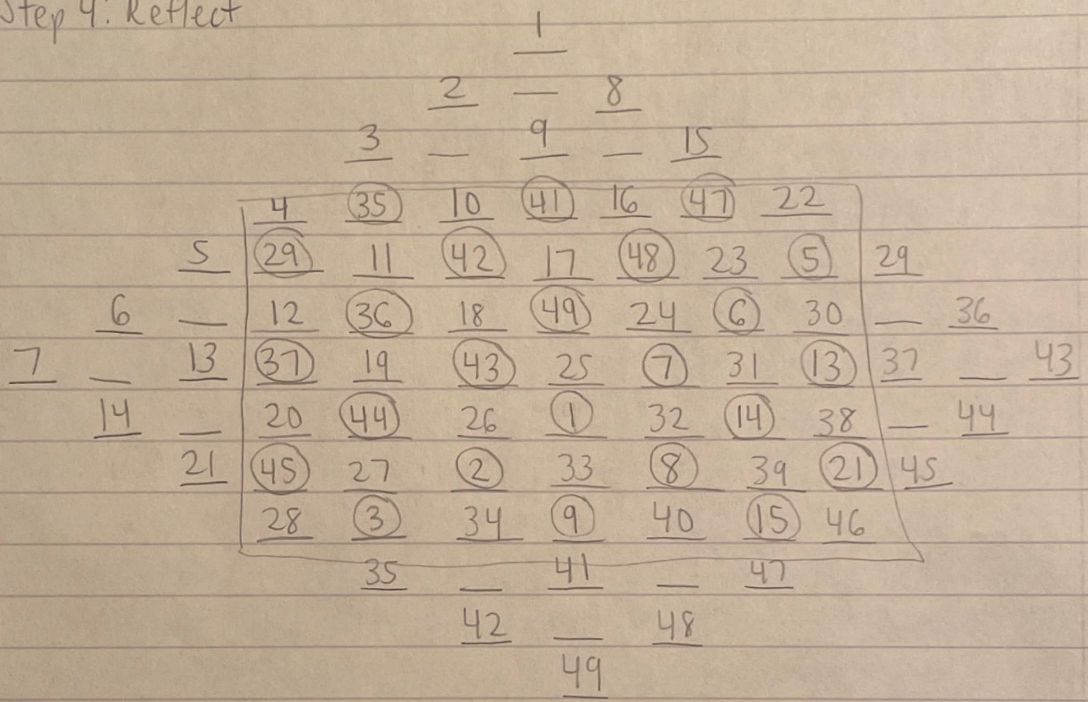
1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

④ Steps 1 & 2: Draw blank 7x7 grid with scaffolding

Step 3: Write numbers on diagonals

Step 4: Reflect



4	35	10	41	16	47	22
29	11	42	17	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

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A↓ | B→	2	4	6
1	B	B	B
3	A	B	B
5	A	A	B
7	A	A	A

They both have an equal chance of winning, so no one is more likely to win. The probability of winning is $\frac{6}{12} = \frac{1}{2}$.

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2	9	4
7	5	3
6	1	8

Deck A: 2, 9, 4
Deck B: 7, 5, 3
Deck C: 6, 1, 8

A↓ | B→	7	5	3
2	B	B	B
9	A	A	A
4	B	B	A

A has probability of winning $\frac{4}{9}$
B has probability of winning $\frac{5}{9}$

B is more likely to win than A

B↓ | C→	6	1	8
7	B	B	C
5	C	B	C
3	C	B	C

B has probability of winning $\frac{4}{9}$
C has probability of winning $\frac{5}{9}$

C is more likely to win than B

A↓ | C→	6	1	8
2	C	A	C
9	A	A	A
4	C	A	C

A has probability of winning $\frac{5}{9}$
C has probability of winning $\frac{4}{9}$

A is more likely to win than C.

Yes, it constitute's a Sucker's Paradox because no single deck is better than all the other ones, as shown by the games above.