

## Homework 21

12/05/2021

$$\begin{array}{l}
 \mu(1) = 1 \quad \mu(2) = -1 \quad \mu(3) = -1 \quad \mu(4) = 0 \\
 \rightarrow \mu(1) = 1 \quad \mu(2) = 0 \quad \mu(3) = -1 \quad \mu(4) = -1 \\
 \mu(5) = -1 \quad \mu(6) = 1 \quad \mu(7) = -1 \quad \mu(8) = 0 \\
 \rightarrow \mu(5) = -2 \quad \mu(6) = -1 \quad \mu(7) = -2 \quad \mu(8) = -2 \\
 \mu(9) = 0 \quad \mu(10) = 1 \quad \mu(11) = -1 \quad \mu(12) = 0 \\
 \rightarrow \mu(9) = -2 \quad \mu(10) = -1 \quad \mu(11) = -2 \quad \mu(12) = -2 \\
 \mu(13) = -1 \quad \mu(14) = 1 \quad \mu(15) = 1 \quad \mu(16) = 0 \\
 \rightarrow \mu(13) = -3 \quad \mu(14) = -2 \quad \mu(15) = -1 \quad \mu(16) = -1 \\
 \mu(17) = -1 \quad \mu(18) = 0 \quad \mu(19) = -1 \quad \mu(20) = 0 \\
 \rightarrow \mu(17) = -2 \quad \mu(18) = -2 \quad \mu(19) = -3 \quad \mu(20) = -3.
 \end{array}$$

2. Goldbach conjecture for  $4 \leq n \leq 30$   
 "Every even whole number greater than 2 is the sum of 2 prime numbers."

$$4 = 2 + 2$$

$$20 = 7 + 13 = 3 + 17$$

$$6 = 3 + 3$$

$$22 = 5 + 17$$

$$8 = 5 + 3$$

$$24 = 5 + 19$$

$$10 = 5 + 5$$

$$26 = 7 + 19$$

$$12 = 5 + 7$$

$$28 = 5 + 23$$

$$14 = 7 + 7$$

$$30 = 7 + 23$$

$$16 = 3 + 13$$

$$18 = 7 + 11$$

3. Twin primes less than 30:

3 and 5, 5 and 7, 11 and 13, 17 and 19

4. Collatz conjecture,  $3n+1$

$$f(n) = \begin{cases} n/2 & n \text{ even} \\ (3n+1)/2 & n \text{ odd} \end{cases}$$

$$2 \rightarrow \frac{2}{2} = 1$$

$$3 \rightarrow \frac{3 \cdot 3 + 1}{2} = 5 \rightarrow \frac{3 \cdot 5 + 1}{2} = 8 \rightarrow \frac{8}{2} = 4 \rightarrow \frac{4}{2} = 2 \rightarrow \frac{2}{2} = 1$$

$$4 \rightarrow \frac{4}{2} = 2 \rightarrow \frac{2}{2} = 1$$

$$5 \rightarrow \frac{3 \cdot 5 + 1}{2} = 8 \rightarrow \frac{8}{2} = 4 \rightarrow \frac{4}{2} = 2 \rightarrow \frac{2}{2} = 1$$

$$6 \rightarrow \frac{6}{2} = 3 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$7 \rightarrow \frac{3 \cdot 7 + 1}{2} = 11 \rightarrow \frac{3 \cdot 11 + 1}{2} = 17 \rightarrow \frac{3 \cdot 17 + 1}{2} = 26 \rightarrow \frac{26}{2} = 13 \rightarrow \frac{3 \cdot 13 + 1}{2} = 20$$

$$20 \rightarrow \frac{20}{2} = 10 \rightarrow \frac{10}{2} = 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \quad (\text{from } 5)$$

$$9 \rightarrow \frac{3 \cdot 9 + 1}{2} = 14 \rightarrow \frac{14}{2} = 7 \rightarrow 11 \rightarrow 17 \rightarrow 26 \rightarrow 13 \rightarrow 20$$

$$\rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$11 \rightarrow 17 \rightarrow 26 \rightarrow 13 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \quad (\text{from } 7)$$

$$12 \rightarrow \frac{12}{2} = 6 \rightarrow 3 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \quad (\text{from } 6)$$

$$13 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \quad (\text{from } 7)$$

$$14 \rightarrow 7 \rightarrow 11 \rightarrow 17 \rightarrow 26 \rightarrow 13 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$15 \rightarrow \frac{3 \cdot 15 + 1}{2} = 23 \rightarrow \frac{3 \cdot 23 + 1}{2} = 35 \rightarrow \frac{3 \cdot 35 + 1}{2} = 53 \rightarrow \frac{3 \cdot 53}{2} = 80$$

$$\rightarrow \frac{80}{2} = 40 \rightarrow \frac{40}{2} = 20 \rightarrow \frac{20}{2} = 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$17 \rightarrow 26 \rightarrow 13 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$18 \rightarrow \frac{18}{2} = 9 \rightarrow 14 \rightarrow 7 \rightarrow 11 \rightarrow 17 \rightarrow 26 \rightarrow 13 \rightarrow 20 \rightarrow 10 \rightarrow 5$$

$$\rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$19 \rightarrow \frac{3 \cdot 19 + 1}{2} = 29 \rightarrow \frac{3 \cdot 29 + 1}{2} = 44 \rightarrow \frac{44}{2} = 22 \rightarrow \frac{22}{2} = 11$$

$$\rightarrow 17 \rightarrow 26 \rightarrow 13 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$20 \rightarrow 10 \rightarrow 5 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$