Sarah Magno
Dr. Z, History of Math
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Homework for Lecture 21 - OK to post

① 1 → square free. $1 = (1 \cdot 1)$ Has 0 prime factors, so $(-1)^0 = \boxed{1} = \mu(0)$
2 → square free. $2 = (2 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(1)$
3 → square free. $3 = (3 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(2)$
4 → NOT square free, since $4 = 2^2$. So $\boxed{0} = \mu(4)$
5 → square free. $5 = (5 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(5)$
6 → square free. $6 = (2 \cdot 3)$. Has 2 prime factors, so $(-1)^2 = \boxed{1} = \mu(6)$
7 → square free. $7 = (7 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(7)$
8 → NOT square free, since $8 = 2^2 \cdot 2$. So $\boxed{0} = \mu(8)$
9 → NOT square free, since $9 = 3^2$. So $\boxed{0} = \mu(9)$
10 → square free. $10 = (5 \cdot 2)$ Has 2 prime factors, so $(-1)^2 = \boxed{1} = \mu(10)$
11 → square free. $11 = (11 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(11)$
12 → NOT square free, since $12 = 2^2 \cdot 3$. So $\boxed{0} = \mu(12)$
13 → square free. $13 = (13 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(13)$
14 → square free $14 = (7 \cdot 2)$ Has 2 prime factors, so $(-1)^2 = \boxed{1} = \mu(14)$
15 → square free. $15 = (5 \cdot 3)$ Has 2 prime factors, so $(-1)^2 = \boxed{1} = \mu(15)$
16 → NOT square free, since $16 = 4^2$. So $\boxed{0} = \mu(16)$
17 → square free $17 = (17 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(17)$
18 → NOT square free, since $18 = 3^2 \cdot 2$. So $\boxed{0} = \mu(18)$
19 → square free. $19 = (19 \cdot 1)$ Has 1 prime factor, so $(-1)^1 = \boxed{-1} = \mu(19)$
20 → NOT square free, since $20 = 2^2 \cdot 5$. So $\boxed{0} = \mu(20)$

For $1 \le n \le 20$, we compute Mertens function as

$$M(n) = \sum_{i=1}^{20} \mu(n) = 1(5) + (-1)(8) = -3$$

(2) $4 = 2+2$ $18 = 13+5$
$6 = 3+3$ $20 = 17+3$
$8 = 3+5$ $22 = 19+3$
$10 = 3+7$ $24 = 19+5$
$12 = 5+7$ $26 = 23+3$
$14 = 3+11$ $28 = 23+5$
$16 = 13+3$ $30 = 23+7$

Thus Goldbach's conjecture holds, since we've shown that each even integer (greater than 2) is the sum of two prime numbers.

(3) (3,5) (5,7) (11,13) (17,19) (29,31)

(4) $2 \to 1$
$3 \to 3(3)+1 = 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$4 \to 2 \to 1$
$5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$6 \to 3 \to 3(3)+1 = 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$7 \to 7(3)+1 = 22 \to 11 \to 11(3)+1 = 34 \to 17 \to 17(3)+1 = 52 \to 26 \to$
$13 \to 13(3)+1 = 40 \to 20 \to 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$8 \to 4 \to 2 \to 1$
$9 \to 9(3)+1 = 28 \to 14 \to 7 \to 7(3)+1 = 22 \to 11 \to 11(3)+1 = 34 \to 17 \to 17(3)+1 = 52 \to$
$26 \to 13 \to 13(3)+1 = 40 \to 20 \to 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$11 \to 11(3)+1 = 34 \to 17 \to 17(3)+1 = 52 \to 26 \to 13 \to 13(3)+1 = 40 \to 20 \to$
$10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$12 \to 6 \to 3 \to 3(3)+1 = 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$13 \to 13(3)+1 = 40 \to 20 \to 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to 2 \to 1$
$14 \to 7 \to 7(3)+1 = 22 \to 11 \to 11(3)+1 = 34 \to 17 \to 17(3)+1 = 52 \to$
$26 \to 13 \to 13(3)+1 = 40 \to 20 \to 10 \to 5 \to 5(3)+1 = 16 \to 8 \to 4 \to$
$2 \to 1$

4 con't $15 \to 15(3)+1=46 \to 23 \to 23(3)+1=70 \to 35 \to 35(3)+1=106 \to$
$53 \to 53(3)+1=160 \to 80 \to 40 \to 20 \to 10 \to 5 \to 5(3)+1=16 \to$
$8 \to 4 \to 2 \to 1$

$16 \to 8 \to 4 \to 2 \to 1$

$17 \to 17(3)+1=52 \to 26 \to 13 \to 13(3)+1=40 \to 20 \to 10 \to 5 \to$
$5(3)+1=16 \to 8 \to 4 \to 2 \to 1$

$18 \to 9 \to 9(3)+1=28 \to 14 \to 7 \to 7(3)+1=22 \to 11 \to 11(3)+1=34 \to$
$17 \to 17(3)+1=52 \to 26 \to 13 \to 13(3)+1=40 \to 20 \to 10 \to 5 \to$
$5(3)+1=16 \to 8 \to 4 \to 2 \to 1$

$19 \to 19(3)+1=58 \to 29 \to 29(3)+1=88 \to 44 \to 22 \to 11 \to 11(3)+1=$
$34 \to 17 \to 17(3)+1=52 \to 26 \to 13 \to 13(3)+1=40 \to 20 \to 10 \to 5 \to$
$5(3)+1=16 \to 8 \to 4 \to 2 \to 1$

$20 \to 10 \to 5 \to 5(3)+1=16 \to 8 \to 4 \to 2 \to 1$

Thus in each case, we arrive at 1 eventually, so Collatz conjecture holds.