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Homework for Lecture 20 - OK to post

(1) Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Pascal's Triangle Mod 2

1
1 1
1 0 1
1 1 1 1
1 0 0 0 1
1 1 0 0 1 1
1 0 1 0 1 0 1
1 1 1 1 1 1 1 1

An even number in Pascal's Triangle becomes 0 in the Mod 2 version
An odd number in Pascal's Triangle becomes 1 in the Mod 2 version

Pascal's Triangle Mod 2 is a fractal because it has self-similarity. The three smaller triangles in red that surround the upside down triangle of 0's in blue are the same.

The fractal diagram is

$$
\begin{array}{ccccc}
A(n) = & & A(n-1) & & \\
& A(n-1) & 0 & A(n-1) &
\end{array}
$$

② i.) The first 10 terms are: 0.5, 0.25, 0.1875, 0.15234375, 0.1291351318, 0.1124592496, 0.0998121667S, 0.08984969812, 0.08177672987, 0.07508929632

In this case, it is tending to a single number. By the formula $\frac{a-1}{a}$, we see that $\frac{1-1}{1}=0$, which confirms our results since the values seem to be tending to 0.

ii.) The first 10 terms are: 0.5, 0.625, 0.5859375, 0.6065368652, 0.5966247409, 0.6016591486, 0.5991635437, 0.600416479, 0.5997913269, 0.6001042277

In this case, it is tending to a single number. By the formula $\frac{a-1}{a}$, we see that $\frac{2.5-1}{2.5}=\frac{3}{5}=0.6$, which confirms our results since the values seem to be tending to that number.

iii.) The first 10 terms are: 0.5, 0.775, 0.5405625, 0.7698995191, 0.5491781737, 0.7675026724, 0.5531711928, 0.7662357552, 0.5552674202, 0.765531088

In this case, it is fluctuating between two numbers. This is because $a=3.1$, which is greater than 3, which is the bifurcation cut-off where it transitions to period 2.

iv.) The first 10 terms are: 0.5, 0.875, 0.3828125, 0.8269348145, 0.5008976948, 0.8749971795, 0.3828199038, 0.8269408877, 0.5008837959, 0.8749972662

In this case, it is fluctuating between four numbers. This is because $a=3.5$, which is greater than 3.449, which is the bifurcation cut-off where it transitions to period 4.

③ In the logistic map $x_{n+1} = kx_n(1-x_n)$ where k is the reproduction constant, when $k<1$, then the population will eventually go extinct. If $1<k<3$, in the long run, the population will stabilize. But beginning at $k>3$, for a while, there will be an ultimate period of 2. Sooner or later, making k even bigger, there will be an ultimate period of 4. Again, sooner or later, making k even bigger, there will be an ultimate period of 8. Thus r_k will be the transition from period 2^{k-1} to period 2^k. Feigenbaum noticed that

$$\lim_{k\to\infty} \frac{r_{k+1}-r_k}{r_k-r_{k-1}}$$

exists and that these ratios tend to the constant 4.6692016.