

Sarah Magno  
Dr. Z, History of Math  
11/28/21

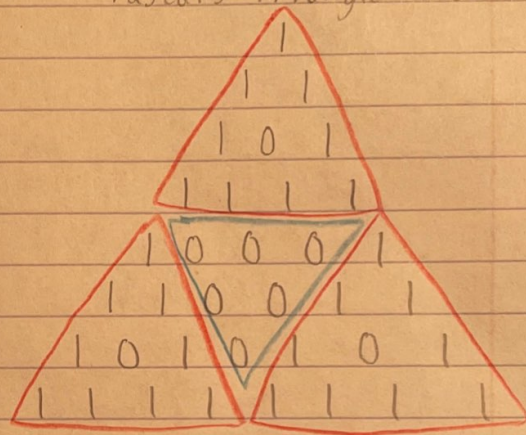
### Homework for Lecture 20 - OK to post

①

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Pascal's Triangle Mod 2



An even number in Pascal's Triangle becomes 0 in the Mod 2 version  
An odd number in Pascal's Triangle becomes 1 in the Mod 2 version

Pascal's Triangle Mod 2 is a fractal because it has self-similarity.  
The three smaller triangles in red that surround the upside down triangle of 0's in blue are the same.

The fractal diagram is

$$A(n) = \begin{matrix} & & A(n-1) & & \\ & A(n-1) & & 0 & & A(n-1) \\ & & & & & & \end{matrix}$$

② i.) The first 10 terms are: 0.5, 0.25, 0.1875, 0.15234375,  
0.1291351318, 0.1124592496, 0.09981216675, 0.08984969812,  
0.08177672987, 0.07508929632

In this case, it is tending to a single number. By the formula  $\frac{a-1}{a}$ ,  
we see that  $\frac{1-1}{1} = 0$ , which confirms our results since the values seem to  
be tending to 0.

ii.) The first 10 terms are: 0.5, 0.625, 0.5859375, 0.6065368652,  
0.5966247409, 0.6016591486, 0.5991635437, 0.600416479,  
0.5997913269, 0.6001042277

In this case, it is tending to a single number. By the formula  $\frac{a-1}{a}$ ,  
we see that  $\frac{2.5-1}{2.5} = \frac{3}{5} = 0.6$ , which confirms our results since  
the values seem to be tending to that number.

iii.) The first 10 terms are: 0.5, 0.775, 0.5405625, 0.7698995191,  
0.5491781737, 0.7675026724, 0.5531711928, 0.7662357552,  
0.5552674202, 0.765531088

In this case, it is fluctuating between two numbers. This is because  
 $a=3.1$ , which is greater than 3, which is the bifurcation cut-off  
where it transitions to period 2.

iv.) The first 10 terms are: 0.5, 0.875, 0.3828125, 0.8269348145,  
0.5008976948, 0.8749971795, 0.3828199038, 0.8269408877,  
0.5008837959, 0.8749972662

In this case, it is fluctuating between four numbers. This is  
because  $a=3.5$ , which is greater than 3.449, which is the bifurcation  
cut-off where it transitions to period 4.

③ In the logistic map  $x_{n+1} = kx_n(1-x_n)$  where  $k$  is the reproduction constant, when  $k < 1$ , then the population will eventually go extinct. If  $1 < k < 3$ , in the long run, the population will stabilize. But beginning at  $k > 3$ , for a while, there will be an ultimate period of 2. Sooner or later, making  $k$  even bigger, there will be an ultimate period of 4. Again, sooner or later, making  $k$  even bigger, there will be an ultimate period of 8. Thus  $r_k$  will be the transition from period  $2^{k-1}$  to period  $2^k$ . Feigenbaum noticed that

$$\lim_{k \rightarrow \infty} \frac{r_{k+1} - r_k}{r_k - r_{k-1}}$$

exists and that these ratios tend to the constant 4.6692016.