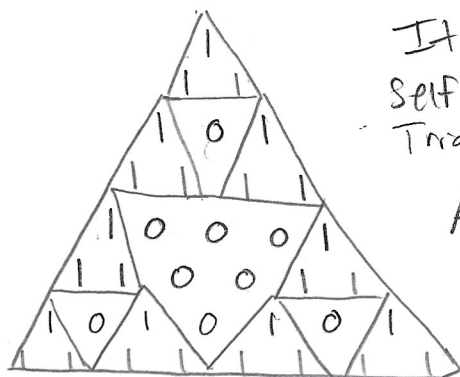


Quin Boob

HW 20

Ok to Post

1)



It is a fractal because it is self-similar forming Sierpinski's Triangle with the form:

$$A(n) = \begin{matrix} A(n-1) \\ A(n-1) & A(n-1) \end{matrix}$$

2) $x_{n+1} = a x_n (1 - x_n) \quad x_0 = 0.5$

i) $a=1$ [0.25, 0.1875, 0.15234..., 0.12913, 0.112455, 0.0998088, 0.089847, 0.0817745, 0.0750874, 0.06944928]

$$\bar{x} = \frac{1-1}{1} = 0$$

ii) $a=2.5$ [0.625, 0.5859, 0.60655, 0.5966177, 0.601662, 0.599162, 0.60041, 0.59979, 0.60010, 0.59994]

$$\bar{x} = \frac{2.5-1}{2.5} = 0.6$$

iii) $a=3.1$ [0.775, 0.54056, 0.7699, 0.54917, 0.7675, 0.5531, 0.76625, 0.5552, 0.76555, 0.55639] - This has a cycle 2 fluctuating between 2 values ≈ 0.765 and ≈ 0.556

$$\text{avg}(\bar{x}) = \frac{3.1-1}{3.1} = 0.677$$

iv) $a=3.5$ [0.875, 0.3828, 0.8269, 0.500977, 0.8749, 0.3836, 0.8270, 0.5007, 0.874, 0.385, 0.828, 0.498] This has a cycle of period 4 with $\bar{x} = 0.874, 0.385, 0.828, 0.498$

$$\text{avg}(\bar{x}) = \frac{3.5-1}{3.5} = 0.7143$$

4) Feigenbaum Constant: $\lim_{n \rightarrow \infty} \frac{r_{n+1} - r_{n+2}}{r_n - r_{n-1}} = 4.6692 \dots$

where r_n are the values, a , where $x_{n+1} = a x_n (1 - x_n)$ bifurcate