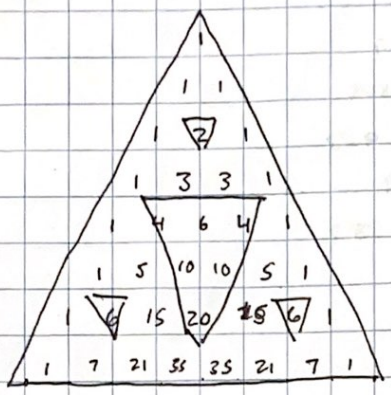
MATH 437 : HW 20

①



It becomes a fractal when we look at the pattern of evens and odds appearing within the triangle. It is a fractal because this pattern will occur on an a smaller / larger scale.

② [ON NEXT PAGE]

③ Two mathematical constants which both express ratios in a bifurcation diagram for a nonlinear map

$\approx 0.4669\,2016\ldots$

$$x_{n+1} = \alpha x_n (1 - x_n)$$

(i) $\alpha = 1$, $x_0 = 0.5$:

$x_1 = 0.25$
$x_2 = 0.1817$
$x_3 = 0.15234375$
$x_4 = 0.129135\,1318$
$x_5 = 0.112454\,2495$
$x_6 = 0.0998\,121667$
$x_7 = 0.089849\,69808$
$x_8 = 0.08177\,672983$
$x_9 = 0.075089\,629$
$x_{10} = 0.069\,450\,893\,87$

TENDS TO SINGLE NUMBER
LIMIT POINTS: 0 (not sure)

(ii) $\alpha = 2.5$; $x_0 = 0.5$

$x_1 = 0.625$
$x_2 = 0.585937$
$x_3 = 0.606536\,8652$
$x_4 = 0.596624\,7409$
$x_5 = 0.601\,659\,1486$
$x_6 = 0.599\,163\,5438$
$x_7 = 0.600\,416\,479$
$x_8 = 0.599\,791\,3269$
$x_9 = 0.600\,104\,2277$
$x_{10} = 0.599\,947\,859$

TENDS TO SINGLE NUMBER
LIMITING POINTS: 0.6 (not sure)

(iii) $\alpha = 3.1$; $x_0 = 0.5$:

$x_1 = 0.775$
$x_2 = 0.540\,625$
$x_3 = 0.769\,899\,5191$
$x_4 = 0.549\,178\,1737$
$x_5 = 0.767\,502\,6724$
$x_6 = 0.553\,171\,1928$
$x_7 = 0.766\,235\,7552$
$x_8 = 0.555\,267\,4202$
$x_9 = 0.765\,531\,088$
$x_{10} = 0.556\,429\,048$

TENDS TO ~~SINGLE NUMBER~~ TWO NUMBERS
LIMITING POINTS: ~~(not sure)~~ 0.6 & 0.7 (not sure)

(iv) $\alpha = 3.5$; $x_0 = 0.5$:

$x_1 = 0.875$
$x_2 = 0.382\,8125$
$x_3 = 0.826\,984\,8144$
$x_4 = 0.500\,897\,6948$
$x_5 = 0.874\,997\,1795$
$x_6 = 0.382\,819\,9027$
$x_7 = 0.826\,940\,8876$
$x_8 = 0.500\,882\,7958$
$x_9 = 0.874\,997\,2661$
$x_{10} = 0.382\,819\,6767$

TENDS TO 3 NUMBERS
LIMITING POINTS: