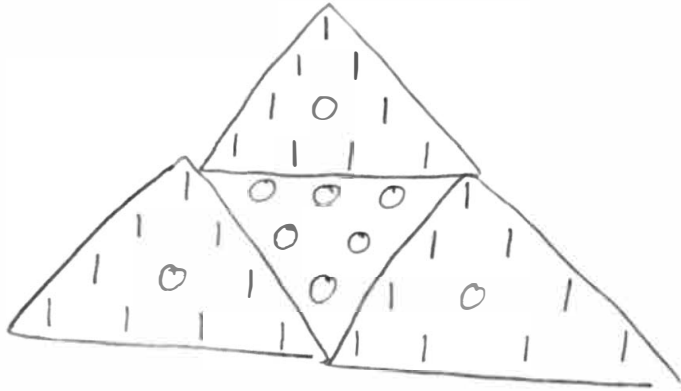


1)



Pascal's triangle
mod 2 [8 rows]

The triangle we drew above is self similar. Since we can build the larger 8 row triangle recursively using the smaller 4 row triangle, it's a fractal. In other words, the triangle above represents

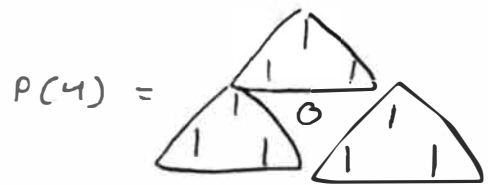
$$P(n) = \begin{matrix} P(n/2) \\ \circ \\ P(n/2) \quad P(n/2) \end{matrix}$$

where n is the # of rows in the triangle

We can see that

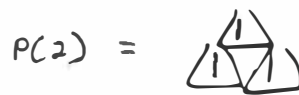
$$P(8) = \begin{matrix} P(4) \\ \circ \\ P(4) \quad P(4) \end{matrix}$$

where



$$\text{or } P(4) = \begin{matrix} P(2) \\ \circ \\ P(2) \quad P(2) \end{matrix}$$

where



$$\text{or } P(2) = \begin{matrix} P(1) \\ \circ \\ P(1) \quad P(1) \end{matrix}$$

where

$$P(1) = 1$$

$P(1) = \text{Base}$

2) $x_{(n+1)} - ax_n(1-x_n), x_0 = 0.5$

(i) $a = 1$

Doesn't look like it's converging to any number.

First 10

```
0.5
0.25
0.1875
0.15234375
0.1291351318359375
0.11245924956165254
0.09981216674968249
0.08984969811841606
0.08177672986644556
0.07508929631879595
```

last 10 from 1000

```
0.0010003251822699275
0.0009993245317996443
0.0009983258822797876
0.000997329227712558
0.0009963345621241083
0.0009953418795644253
0.0009943511741072105
0.000993362439849762
0.0009923756709128578
0.0009913908614406382
```

(ii) $a = 2.5$

Tends towards a single number.
The limiting point seems to be 0.6

```
0.5
0.625
0.5859375
0.606536865234375
0.5966247408650815
0.6016591486318896
0.5991635437485985
0.6004164789780495
0.5997913268741273
0.6001042277017528
```

```
0.6000000000000001
0.6
0.6000000000000001
0.6
0.6000000000000001
0.6
0.6000000000000001
0.6
0.6000000000000001
0.6
```

(iii) $a = 3.1$

Tends towards 2 numbers, the limiting points seems to cycle between (0.7645665...) and (0.558014...)

```
0.5
0.775
0.5405625000000001
0.769899519140625
0.5491781736597441
0.7675026724300255
0.5531711927526629
0.7662357552099034
0.5552674202082185
0.7655310880169375
```

```
0.7645665199585945
0.5580141252026958
0.7645665199585945
0.5580141252026958
0.7645665199585945
0.5580141252026958
0.7645665199585945
0.5580141252026958
0.7645665199585945
0.5580141252026958
```

(iv) $a = 3.5$

Tends towards 4 numbers, the limiting points seem to be (0.50088...), (0.874997...), (0.382819...), (0.826940...)

```
0.5
0.875
0.3828125
0.826934814453125
0.5008976948447526
0.87499717950388
0.3828199037744718
0.826940887670016
0.500883795893397
0.8749972661668659
```

```
0.8269407065914387
0.5008842103072179
0.8749972636024641
0.38281968301732416
0.8269407065914387
0.5008842103072179
0.8749972636024641
0.38281968301732416
0.8269407065914387
0.5008842103072179
```

$$2) \quad x_{n+1} = a x_n (1 - x_n)$$

$$\cancel{x_0 = 0.5}$$

$$(i) \quad \cancel{a = 1}$$

3) Define the Feigenbaum constant

We can see that when a is greater than 3 but less than 3.4494897, the period is 2. [period length is the number of unique values of x_n (when n is large) that it cycles]. Let a_n be the bifurcation parameter at which the period changes from 2^{n-1} to 2^n

$$\text{Then } a_1 = 3$$

$$a_2 = 3.4494897$$

$$a_3 = 3.5440903$$

$$a_4 = 3.5644073$$

⋮

And the Feigenbaum constant is

$$\lim_{n \rightarrow \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \quad \text{which is equal to } \boxed{4.669201609}$$