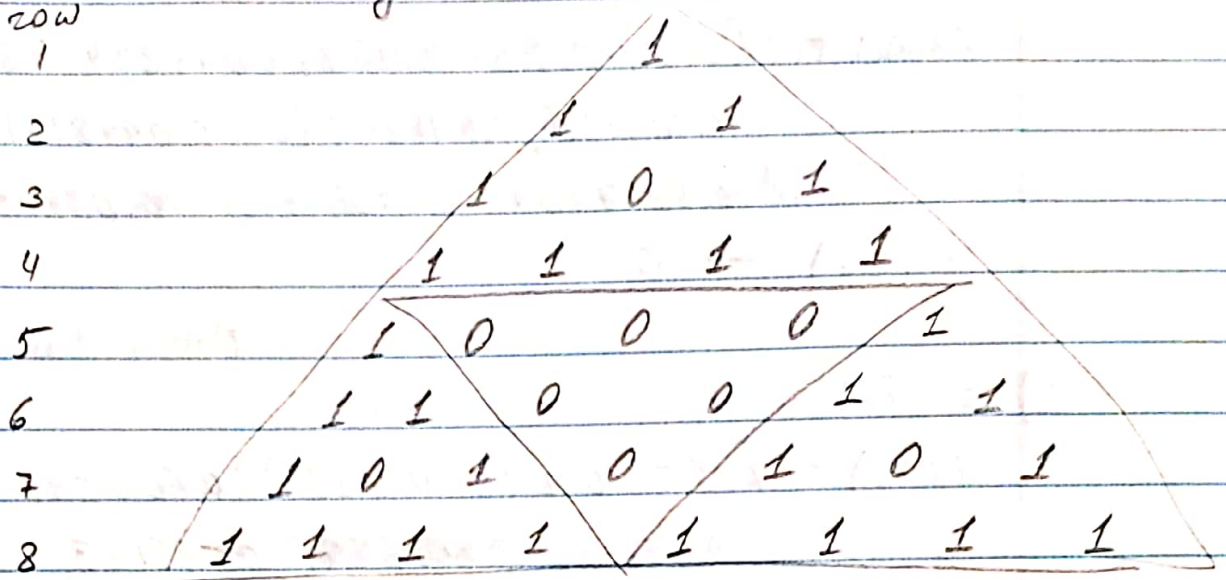


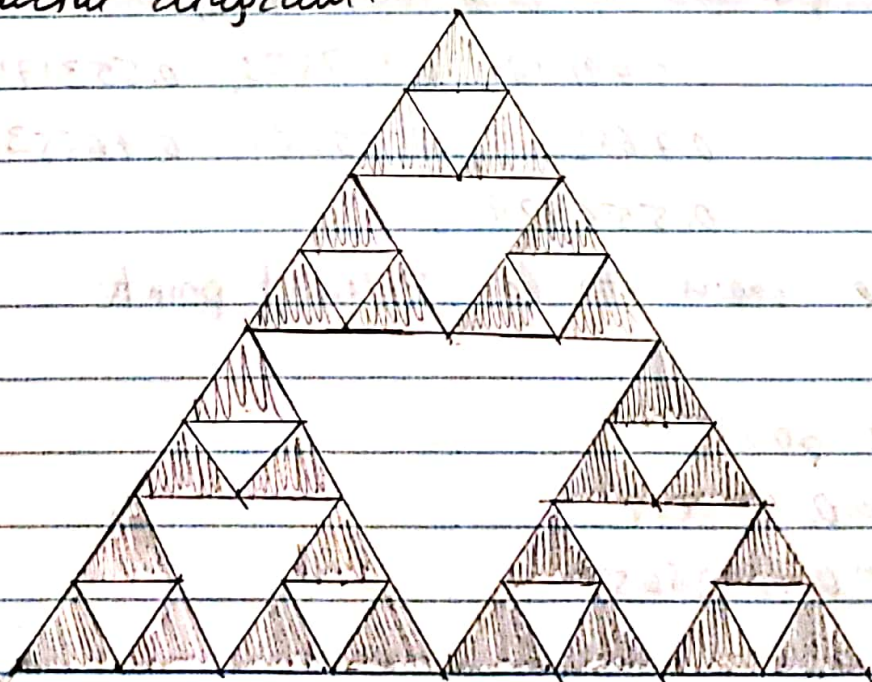
Homework 20

11/28/2021

1. Pascal's Triangle modulo 2



A fractal is a never ending pattern. Pascal's triangle mod 2 consists of repeating triangles with a similar. The basic pattern is in the first 3 rows and this pattern continues to repeat.
Fractal diagram:



$$2. \quad x_{n+1} = a x_n (1 - x_n) \quad \text{with } x_0 = 0.5$$

$$i) \quad a = 1$$

$$f(x_n) = [0.5, 0.25, 0.1875, 0.15234375, \\ 0.129135, 0.1124592, 0.099812167, \\ 0.0817767, 0.0750393, 0.069451 \dots]$$

$$f(x_n) \rightarrow 0$$

$$ii) \quad a = 2.5$$

$$f(x_n) = [0.5, 0.625, 0.5859, 0.60654, \\ 0.5966, 0.601659, 0.599163, \\ 0.6004164, 0.59979, 0.6001042, \\ 0.5999478 \dots]$$

$$\text{the limit is: } \frac{2.5 - 1}{2.5} \approx \boxed{0.6}$$

$$iii) \quad a = 3.1$$

$$f(x_n) = [0.5, 0.775, 0.54056, 0.769899, \\ 0.549178, 0.767503, 0.5531719, \\ 0.766236, 0.555267, 0.765531, \\ 0.556429]$$

0.209102

There seem to be 2 limit points:

Limit points:

$$\approx 0.558014$$

$$\approx 0.7645665$$

iv) $q = 3.5$

$$f(x_n) = [0.5, 0.875, 0.3828125, 0.8269348, \\ 0.50089769, 0.874997179, 0.3828199, \\ 0.82694088767, 0.50088379, 0.874997266, \\ 0.382819676].$$

There seem to be 4 limit points.

Limit points

$$\approx 0.50088$$

$$\approx 0.874997$$

$$\approx 0.38281968$$

$$= 0.8269407$$

3. The famous Feigenbaum constant represents the ratio between the diameters of successive circles on the real axis in the complex plane. It is a universal constant for fractions approaching chaos.