1)

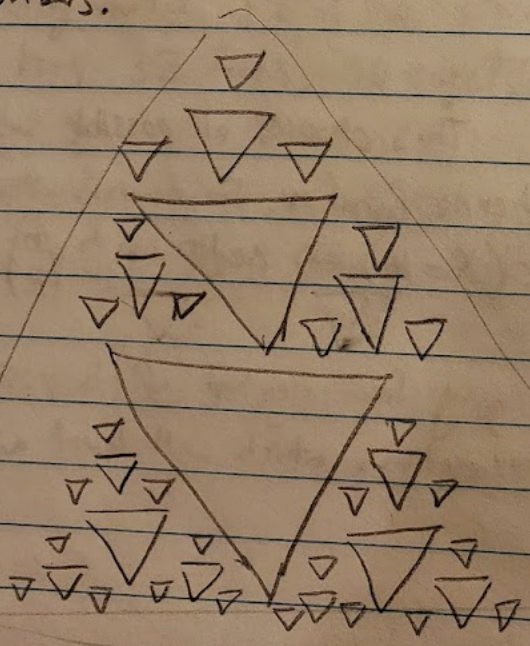
HW 20 ok to post

1)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

we do mod 2 and see the even # turn into 0's

It is fractal because it has the identical pattern in each of the 3 corners which surround a triangle of even numbers.



2)

(i) a = 1

f(0) = 0.5
f(1) = 0.25
f(2) = 0.1875
f(3) = 0.15234375
f(4) = 0.1291351318359375
f(5) = 0.11245924956165254
f(6) = 0.09981216674968249
f(7) = 0.08984969811841606
f(8) = 0.08177672986644556
f(9) = 0.07508929631879595
f(10) = 0.06945089389714401

Seems to converge to one point.

(ii) a = 2.5

f(0) = 0.5
f(1) = 0.625
f(2) = 0.5859375
f(3) = 0.606536865234375
f(4) = 0.5966247408650815
f(5) = 0.6016591486318896
f(6) = 0.5991635437485985
f(7) = 0.6004164789780495
f(8) = 0.5997913268741273
f(9) = 0.6001042277017528
f(10) = 0.599947858990589

Seems to converge at an orbit of length 2

(iii) a = 3.1

f(0) = 0.5
f(1) = 0.775
f(2) = 0.5405625000000001

f(3) = 0.769899519140625
f(4) = 0.5491781736597441
f(5) = 0.7675026724300255
f(6) = 0.5531711927526629
f(7) = 0.7662357552099034
f(8) = 0.5552674202082185
f(9) = 0.7655310880169375
f(10) = 0.5564290480192784

Seems to converge at an orbit of length 2

(iv) a = 3.5

f(0) = 0.5
f(1) = 0.875
f(2) = 0.3828125
f(3) = 0.826934814453125
f(4) = 0.5008976948447526
f(5) = 0.87499717950388
f(6) = 0.3828199037744718
f(7) = 0.826940887670016
f(8) = 0.500883795893397
f(9) = 0.8749972661668659
f(10) = 0.38281967628581853

Seems to converge at an orbit of length 3

3)

3) Feigenbaum Constant

1) δ is the limiting ratio of each bifurication interval to the next between every period doubling, of one-parameter map $x_{i+1} = f(x_i)$ where $f(x)$ is a function parameterized by the bifurication parameter a.

It is given by the limit

$$\delta = \lim_{n\to\infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.669$$

where a_n are discrete values of a at the n^{th} period doubling.

2) alpha constant

$$\alpha = 2.502907\ldots$$

is the ratio between the width of a tine and the width of one of its two subtines.

A negative sign is applied to α when the ratio between the lower subtine and the width of the tine is measured.