HOMEWORK #2 - NINA CHALGERI

① $\frac{(1+2+3+\ldots+n^2)}{n}$

PROOF.

We add up all the numbers that appear in the magic square because each number will be there at least once. Then, we take the sum and divide it by n because the ~~product~~ magic constant needs to be divisible by the dimensions of the square (n).

② $3\times3 \Rightarrow 3^2 = 9 \quad 1+2+3+4+5+6+7+8+9 = 45 \Rightarrow 45/5 = 15$

1 5 9 = 15

$9+4+2 = 15$

$7+6+2 = 15$

$5+7+3 = 15 \Rightarrow 2+$

$8+3+4 = 15$

$$\begin{bmatrix} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{bmatrix}$$

③ $4\times4 \Rightarrow$ magic constant: 34

1			4
	6	7	
	10	11	
13			16

$\Rightarrow$

$$\begin{bmatrix} 1 & 15 & 14 & 4 \\ 12 & 6 & 7 & 9 \\ 8 & 10 & 11 & 5 \\ 13 & 3 & 2 & 16 \end{bmatrix}$$

④ $7\times7 \Rightarrow 7^2 = 49 \Rightarrow$ magic constant $\Rightarrow 175$

$\Rightarrow$

$$\begin{bmatrix} 22 & 47 & 16 & 41 & 10 & 35 & 4 \\ 5 & 23 & 48 & 17 & 42 & 11 & 29 \\ 30 & 6 & 24 & 49 & 18 & 36 & 12 \\ 13 & 31 & 7 & 25 & 43 & 19 & 37 \\ 38 & 14 & 32 & 1 & 26 & 44 & 20 \\ 21 & 39 & 8 & 33 & 2 & 27 & 45 \\ 46 & 15 & 40 & 9 & 34 & 3 & 28 \end{bmatrix}$$

⑤ PA: 1, 3, 5, 7
PB: 2, 4, 6

~~1<2~~
3>2
~~3<4~~
5>4

A	B	A	B	A	B
1 < 2		1 < 4		1 < 6	
3 > 2		3 < 4		3 < 6	
5 > 2		5 > 4		5 < 6	
7 > 2		7 > 4		7 > 6	

4 × 3 = 12 total outcomes

PA has a ~~4/5~~ 1/2 chance of winning

⇒ PB has a 1/2 chance of winning

∴ Neither are more likely to win

· P(winning) = 1/2 for both PA and PB

⑥

$$\begin{bmatrix} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{bmatrix} \begin{matrix} \rightarrow \text{DECK A} \\ \rightarrow \text{DECK B} \\ \rightarrow \text{DECK C} \end{matrix}$$

· A vs B:

A	B	A	B	A	B
2 < 7		9 > 7		4 < 7	
2 < 5		9 > 5		4 < 5	
2 < 3		9 > 3		4 > 3	

9 outcomes

4/9 chance deck A wins ⇒ Deck B

· B vs C:

7 > 6	5 < 6	3 < 6
7 > 1	5 > 1	3 > 1
7 < 8	5 < 8	3 < 8

9 outcomes ⇒ DECK C

· A vs C

2 < 6	9 > 6	4 < 6
2 > 1	9 > 1	4 > 1
2 < 8	9 > 8	4 < 8

9 outcomes ⇒ DECK A

P(A losing) = 3/9 = 1/3