

Homework II

- ① All the numbers in an $n \times n$ magic square range from 1 to n^2 with NO duplicates

We know that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, so $\sum_{i=1}^{n^2} i = \frac{n^2(n^2+1)}{2}$

There are n rows (or n columns), so we divide this value by n to get the sum of each row or column

$$\frac{n^2(n^2+1)}{2n} = \frac{n(n^2+1)}{2} \text{ is the sum of each row or column}$$

Proof by Induction:

base case $n=1$:

$\boxed{1}$ is the 1×1 magic square

$$\frac{1(1+1)}{2} = 1 \checkmark$$

Inductive step:

Assuming it is true for $n-1$,

$$\frac{(n-1)((n-1)^2+1)}{2},$$

It is also true for n

$$\frac{(n-1+1)((n-1+1)^2+1)}{2} = \frac{n(n^2+1)}{2} \checkmark$$

② 3×3 magic square:

9	1	5
3	4	8
6	7	2

③ 4×4 magic square:

16	9	2	13
5	10	11	8
9	6	7	12
4	15	14	1

④ 7×7 magic square:

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

⑤

	2	4	6	B
1	B	B	B	
3	A	B	B	
5	A	A	B	
7	A	A	A	
A				

Person A and person B are both equally likely to win, each with a probability of $\frac{6}{12}$ or $\frac{1}{2}$

⑥

9	1	5	- A
3	4	8	- B
6	7	2	- C

A vs B:

	3	4	8
9	A	A	A
1	B	B	B
5	A	A	B

} A is more likely to win

B vs C:

	6	7	2
3	C	C	B
4	C	C	B
8	B	B	B

} B is more likely to win

A vs C:

	6	7	2
9	A	A	A
1	C	C	C
5	C	C	A

} C is more likely to win

This does constitute a sucker's paradox because there is no "best" deck. All decks are worse than one and better than another