

Homework for Lecture 2.

1. An $n \times n$ magic square contains integer from $1 - n^2$

$$S(n) = \frac{(1+n^2) \cdot n^2}{2} \quad (\text{Gauss theory})$$

Because each row and column add to the same amount, there is a column

$$P(n) = S(n)/n = \frac{(1+n^2) \cdot n}{2}$$

2.
$$\begin{array}{ccc} 1 & & \\ 2 & 4 & 9 \\ 3 & 5 & 7 \\ & 6 & 8 \\ & 9 & \end{array} \Rightarrow \begin{array}{ccc} & 9 & 4 \\ & 7 & 5 \\ & 6 & 1 \\ & 8 & \end{array}$$

3.
$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \Rightarrow \begin{array}{cc} 1 & 4 \\ 6 & 7 \\ 10 & 11 \\ 13 & 16 \end{array} \Rightarrow \begin{array}{cccc} 1 & 15 & 14 & 4 \\ 6 & 7 & 8 & \\ 9 & 10 & 11 & 13 \\ 13 & 3 & 2 & 16 \end{array}$$

4.
$$\begin{array}{cccc} 1 & & & \\ 2 & 8 & & \\ 3 & 9 & 15 & 16 \\ 4 & 10 & 11 & 22 \\ 5 & 11 & 17 & 23 \\ 6 & 12 & 18 & 24 \\ 7 & 13 & 19 & 25 \\ 8 & 14 & 20 & 26 \\ 9 & 15 & 21 & 27 \\ 10 & 16 & 22 & 28 \\ 11 & 17 & 23 & 29 \\ 12 & 18 & 24 & 30 \\ 13 & 19 & 25 & 31 \\ 14 & 20 & 26 & 32 \\ 15 & 21 & 27 & 33 \\ 16 & 22 & 28 & 34 \\ 17 & 23 & 29 & 35 \\ 18 & 24 & 30 & 36 \\ 19 & 25 & 31 & 37 \\ 20 & 26 & 32 & 38 \\ 21 & 27 & 33 & 39 \\ 22 & 28 & 34 & 40 \\ 23 & 29 & 35 & 41 \\ 24 & 30 & 36 & 42 \\ 25 & 31 & 37 & 43 \\ 26 & 32 & 38 & 44 \\ 27 & 33 & 39 & 45 \\ 28 & 34 & 40 & 46 \\ 29 & 35 & 41 & 47 \\ 30 & 36 & 42 & 48 \\ 31 & 37 & 43 & 49 \end{array} \Rightarrow \begin{array}{cccc} 2 & 4 & 10 & 35 \\ 5 & 3 & 18 & 11 \\ 30 & 6 & 24 & 18 \\ \Rightarrow 13 & 31 & 7 & 25 \\ 38 & 14 & 32 & 1 \\ 31 & 39 & 8 & 33 \\ 46 & 15 & 40 & 9 \\ 34 & 3 & 28 & \end{array}$$

5. A 1 1 3 5 7

B 1 2 4 6

2 1 B A A A



4 | B B A A

6 | B B B A

$$P(B \text{ win}) = 6/12 = 1/2$$

Therefore $P(A \text{ win}) = 1/2$. They are equally likely to win

6. 3×3 magic square

2 9 4

7 5 3

6 1 8

(I) A | 2 4 9

$$P(A \text{ win}) = 4/9$$

B | 3 5 7

$$P(B \text{ win}) = 1 - 4/9 = 5/9$$

2 | B B B

4 | A B B

9 | A A A

(II) B | 3 5 7

$$P(B \text{ win}) = 4/9$$

C | 1 6 8

$$P(C \text{ win}) = 1 - 4/9 = 5/9$$

3 | B C C

5 | B C C

7 | B B C

(III) A | 2 4 9

$$P(A \text{ win}) = 5/9$$

C | 1 6 8

$$P(C \text{ win}) = 1 - 5/9 = 4/9$$

2 | A C C

4 | A C C

9 | A A A

