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OK

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Homework 19

1\. a) $P(\text{at least one } 1) = 1 - \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{25}{36} = \frac{11}{36}$

b) $P(\text{at least one } 1 \text{ in } 10 \text{ rolls}) = 1 - \left(\frac{5}{6}\right)^{10} = 0.84$

c) $P(\text{at least one } 1 \text{ in } n \text{ rolls}) = 1 - \left(\frac{5}{6}\right)^{n}$

2\. Proof of binomial coefficient:

First, consider how we would choose k elements from n when order matters. There are n ways to pick the first, $n-1$ to pick the second and $n-k+1$ to pick the k^{th}.

so: there are

$$n(n-1)(n-2)\ldots(n-k+1) = \frac{n!}{(n-k)!} \text{ ways}$$

to pick an ordered list.

But for us, order doesn't matter. Since each ordered list has k elements, there are $k!$ lists with the same elements. Therefore, we are counting each list $k!$ times. Since we want to count it ones, divide the number of order lists by the number of repeats and:

$$\underbrace{\binom{n}{k} = \frac{\frac{n!}{(n-k)!}}{k!} = \frac{n!}{(n-k)!\,k!}}.$$

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3. Proof of $\binom{n}{k} p^k (1-p)^{n-k}$.

Consider tossing a coin n times, with probability p of getting heads and $1-p$ of getting tails. Then the probability that the first k tosses will land on a head and all the others will land on a tail is given by:

$$\underbrace{p \cdot p \cdot p \ldots \cdot p}_{k \text{ times}} \cdot \underbrace{(1-p)\cdot(1-p) \ldots (1-p)}_{(n-k) \text{ times}}.$$

Since order doesn't matter for us because we just want to get k heads somewhere in the n tosses, there are $\binom{n}{k}$ ways to arrange our outcomes.

Thus $\binom{n}{k} p^k (1-p)^{n-k}$ describes all the cases of getting k heads in n tosses.

4. Proof that: $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = 1$

Transform $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ into

$g(x,y) = f(x) f(y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2}$. Then we can convert to polar coordinates where $r^2 = x^2+y^2$ then our integral becomes:

$$\int_0^\infty \int_0^{2\pi} r \frac{1}{2\pi} e^{-r^2/2} \, d\theta \, dr \quad \text{where} \quad \begin{matrix} r: (0,\infty) \\ \theta: (0, 2\pi) \end{matrix}$$

$$= \int_0^\infty 2\pi r \frac{1}{2\pi} e^{-r^2/2} dr$$

$$= \int_0^\infty r e^{-r^2/2} dr$$

$$= \left(-2 \cdot \tfrac{1}{2} e^{-r^2/2}\right)\Big|_0^\infty = -[e^{-\infty/2} - e^{0/2}] = 1. //$$

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