Sarah Catovic (ok to post) Homework 19 11/21/2021

1\. a) $6 \times 6 = 36$ possibilities when rolling a fair dice twice. You can get a 1 on the first roll, second roll, or on both rolls. There are 5 events that you can get a 1 on the first roll, but not the second, 5 events that you can get a 1 on the second roll, but not the first, and 1 event that you can get a 1 on the second and first roll. Adding those up gets you 11 events and put it over the total number of events, 36, getting $^{11}/_{36}$.

b) 1 − probability (no ones) $= 1 - \frac{5^{10}}{6^{10}}$
because it is as though you are rolling a die missing one choice

c) 1 − probability (no ones) $= 1 - \frac{5^n}{6^n}$

2\. Let's say you're trying to choose a certain number of people from a group of n people. For your first choice, you'd have n possibilities, for the second: $n-1$ possibilities, third: $n-2$ possibilities, etc. So, that would be $(n)(n-1)\cdot(n-2)\dots = n!$. However, since you only want the first k choices, you must divide by $(n-k)!$. That way, you'd have the possibilities of only a k number of choices. Additionally, order does not matter, so we must divide by $k!$, which is the number of ways of ordering the choices. So you get the formula $\frac{n!}{(n-k)!\,k!}$

3\. Each coin toss is a separate event, so each probability would be multiplied to get the total for n tosses. We want k Heads and $n-k$ Tails, so that gives $p^k(1-p)^{n-k}$ as the probability. Since you want to see the possibilities of this happening, it is like choosing this to happen, which is why it is multiplied by the combination theorem.

Prove that

4. $\frac{1}{\sqrt{2\pi}} e^{-x^2/2} = 1$

Let $c = \int_{-\infty}^{\infty} e^{\frac{x^2}{2}} dx$ so $c^2 = \int_{-\infty}^{\infty} e^{-x^2/2} dx \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} dy$
$= \int_{\mathbb{R}^2} e^{\left(\frac{-x^2+y^2}{2}\right)} d(x,y)$

Convert to polar coordinates: $x^2 + y^2 = r^2$ $\quad d(x,y) = r\, d(r,\theta)$
$c^2 = \int_0^{2\pi} \int_0^{\infty} r e^{-\frac{r^2}{2}} dr\, d\theta$

Let $u = \frac{r^2}{2}$ $\quad du = r\,dr$ $\quad$ so $c^2 = \int_0^{2\pi} \int_0^{\infty} e^{-u} du\, d\theta =$
$\int_0^{2\pi} \left(1 - e^{-u}\right)\Big|_0^{\infty} d\theta = \int_0^{2\pi} 1\, d\theta = 2\pi$ $\quad$ so $c^2 = 2\pi \Rightarrow c = \sqrt{2\pi}$