Quin Buob

HW 19

Ok to post

1) a) Pr (at least One 1 from 2 Dice roles) $= 1 - Pr(\text{no } 1) = 1 - (\frac{5}{6})(\frac{5}{6}) = 30.56\,\%$

b) 10 Dice rolls: Pr(at least 1 one) $= 1 - (5/6)^{10} \approx 84\,\%$

b) n Dice rolls: Pr(at least 1 one) $= 1 - (5/6)^n$

2) n independent events

$f(x) = (1+x)^n$

$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \ldots + f^k(0)\frac{x^k}{k!}$

$f(0) = 1$

$f'(x) = n(1+x)^{n-1} \quad f'(0) = n$

$f''(x) = n(n-1)(1-x)^{n-2} \Rightarrow f''(0) = n(n-1)$

$\vdots$

$f^k(x) = n(n-1)(n-2)\ldots(n-k-1)(1-x)^{n-k} \Rightarrow f^k(0) = n(n-1)\ldots(n-k+1)$

$(1+x)^n = 1 + n + n(n-1)\frac{x^2}{2!} + \ldots \frac{x^k}{k!}(n-k+1) = \frac{n!}{k!(n-k)!} = \binom{n}{k}$

3) $pr(H) = p$, $pr(T) = 1-p$

Probability of getting K heds will be p^k, and probability of getting the rest tails is $(1-p)^{n-k}$

Number of heads (k) given the total number of tosses n is $\binom{n}{k}$

From these the probability of of getting K heads will be the probability that you will get heads K times, p^k, times the total number of ways to get heads $\binom{n}{k}$ times the probability you will get tails ~~exacth~~ the rest of the coin tosses, $(1-p)^{n-k}$:

$$Pr(\text{K heads in n tosses}) = \binom{n}{k} p^k (1-p)^k$$

$$I = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

$$I^2 = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

$$I^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2} dy\,dx$$

Convert to polar coordinates

$$I^2 = \frac{1}{2\pi} \int_0^{\infty}\int_0^{2\pi} r e^{-r^2/2} d\theta\,dr$$

$$I^2 = \frac{\cancel{2\pi}}{\cancel{2\pi}} \int_0^{\infty} r e^{-r^2/2} dr$$

$$u = -r^2/2$$
$$du = -r\,dr$$

$$I^2 = \int_0^{\infty} e^{-u} du$$

$$I^2 = -e^{u}\Big]_0^{\infty} = -\cancel{e^{-\infty}}^{\,0} - (-e^0) = 1$$

$$I^2 = 1$$

$$I = 1$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1$$

QED