MATH 437: HOMEWORK 19

① (a) $\frac{11}{36}$

(b) $P(\text{rolling } 1)^{-1} = \frac{5}{6}$ per roll

$\Rightarrow \left(\frac{5}{6}\right)^{10} \Rightarrow 1-\left(\frac{5}{6}\right)^{10} \approx 0.8385$

(c) $1-\left(\frac{5}{6}\right)^{n}$

② ~~You have options~~ When choosing a set of k elements ~~from~~ ~~to~~ out of a set of n elements, the first pick will be ~~from~~ a choice of n elements. Without replacement, the next choice will be from a set of $(n-1)$ elements all the way until you are left with $(n-(k-1))$, after k picks.

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}\cdot\frac{(n-k)!}{(n-k)!} = \frac{n!}{k!(n-k)!}$$

③ If k is the number of successes then $n-k$ is the number of failures. ~~We assume~~ p represents the probability of landing heads and the complement is $1-p$ which is ~~then~~ $P(T)$. Out of n tosses, we want k ~~such~~ successes, this is n choose k or $\binom{n}{k}$. The probability of getting k successes is p^k and the probability of getting failures is $(p-1)^{n-k}$. Therefore,

$$\binom{n}{k} p^k (1-p)^{n-k}$$

④ $$C=\int_{-\infty}^{\infty} e^{-x^2/2}\,dx \Rightarrow C^2=\left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}\,dy\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy$$

POLAR COORDINATES

$$C^2=\int_0^{\infty}\int_0^{2\pi} re^{-r^2/2}\,d\theta\,dr = 2\pi\int_0^{\infty} re^{-r^2/2}\,dr \Rightarrow -e^{r^2/2}\Big|_0^{\infty} = -e^{-\infty}+e^{0}=1$$

$$\Rightarrow C^2 = 2\pi$$

$$C=\sqrt{2\pi}$$