Homework 19

① (a)

Die B \ Die A	1	2	3	4	5	6
1	[1,1]	[2,1]	[3,1]	[4,1]	[5,1]	[6,1]
2	[1,2]	X	X	X	X	X
3	[1,3]	X	X	X	X	X
4	[1,4]	X	X	X	X	X
5	[1,5]	X	X	X	X	X
6	[1,6]	X	X	X	X	X

36 possible outcomes

11 total that have atleast one 1

$= \frac{11}{36}$ or $1 - \left(\frac{5}{6}\right)^2 = 0.3055$

(b) Since doing it the above way would be too tedious for 10 dice, we use math.

probability of not rolling a 1: $\frac{5}{6}$ per roll

$\left(\frac{5}{6}\right)^{10} = 0.1615$

probability of rolling a 1 $= 1 - 0.1615 = 0.8385$

(c) $1 - \left(\frac{5}{6}\right)^n$

② If you are choosing k elements out of n elements, you have n options the first time, $n-1$ the second time, $n-2$ the third time, up until $(n-(k-1))$ the k^{th} time.

This is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-k+1)$

Each group of k items is accounted for $k!$ times

$$\frac{n(n-1)(n-2)(n-3)\cdots(n-k+1)}{k!} \times \left(\frac{(n-k)!}{(n-k)!}\right) = \frac{n!}{k!(n-k!)}$$

③ If getting n is a success, you want k successes, so $n-k$ failures. probability of k successes is p^k, and pobability of $n-k$ failures is $(1-p)^{n-k}$. There are $\binom{n}{k}$ ways to get k successes in n trials

$$= \binom{n}{k}(p^k)(1-p)^{n-k}$$

(4) Let $C = \int_{-\infty}^{\infty} e^{-x^2/2} dx$

x can be replaced by y (dummy variable) so

$$C = \int_{-\infty}^{\infty} e^{-y^2/2} dy.$$

$$C^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2} dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2} dx\,dy$$

polar coordinates:

$$C^2 = \int_0^{\infty}\int_0^{2\pi} re^{-r^2/2} d\theta dr$$

$$= \int_0^{\infty} re^{-r^2/2}(2\pi)dr = 2\pi \int_0^{\infty} re^{-r^2/2} dr$$

$$\rightarrow -e^{-r^2/2}\Big]_0^{\infty} = -e^{-\infty} - -e^{0} = 1$$

$$C^2 = 2\pi$$
$$C = \sqrt{2\pi}$$