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H.W 19

1a. roll a die twice, what is the probability of at least one 1 shows. $1 - \text{Prob}(\text{no ones})$

$= 1 - (5/6)(5/6) = 11/36$

1b. roll a die 10 tens, the probability of at least one 1 shows is $1 - \text{Prob}(\text{no ones})$

$= 1 - (5/6)^{10} = \frac{50700551}{60466176}$

1c. roll a die n times, the probability of at least one 1 show is $1 - (5/6)^n$

2. Let $f(x) = (1+x)^n$

$f'(x) = n(1+x)^{n-1}$, $f'(0) = n$

$f''(x) = n \cdot n-1 \cdot (1+x)^{n-2}$, $f''(0) = n \cdot n-1$

$f^{k}(x) = n \cdot (n-1)(n-2) \ldots (n-k+1)(1+x)^{n-k}$, $f^{k}(0) = n \cdot (n-1) \cdot \ldots \cdot (n-k+1)$

$(1+x)^n = 1 + nx + \frac{n \cdot (n-1)}{2!} \cdot x^2 + \frac{n \cdot (n-1)(n-2)}{3!} x^3 + \ldots + \frac{n \cdot (n-1) \cdot \ldots \cdot (n-k+1)}{k!} x^k$

$= \frac{n!}{k!(n-k)!}$

3. Suppose the coin is Loaded. Prob(H) = P, Prob(T) = 1 - P
Before all the outcomes were equal: $(1/2)^n$
but now Prob of getting k heads is
$\binom{n}{k} p^k \cdot (1-p)^{n-k}$

4. $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} dx = I$

$I \cdot I = \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\right)\left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-y^2/2} dy\right)$

$I^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}} dx\, dy$

Convert to polar cordinates
$-x^2 - y^2 = -(x^2+y^2) = -r^2$
$dx\,dy = r\,dr\,d\theta$

$I^2 = \frac{1}{2\pi} \int_0^{\infty} \int_0^{2\pi} e^{-r^2/2} r\, d\theta\, dr$

$I^2 = \frac{1}{2\pi} \int_0^{\infty} \left(e^{-r^2/2} r\theta \Big|_0^{2\pi} \right) dr$

$I^2 = \int_0^{\infty} e^{-r^2/2} r\, dr$

let $u = -r^2/2$ $du = -r$

$-\int_0^{\infty} e^u du = -e^{-r^2/2} \Big|_0^{\lim_{r \to \infty}} = 0 - (-1) = 1$

$I^2 = 1$

$I = 1$