History of Math	Homework 19	Karon Amin

1) a) $1 - P\{\text{Both don't show } 1\} = 1 - \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{25}{36} = 0.31$

b) $1 - \frac{5^{10}}{6^{10}} = 0.838$

c) $1 - \frac{5^n}{6^n}$

2) We have n items and k spots. In the first spot we can have n items to choose from. In the second spot we have $(n-1)$ items to choose from. In the kth spot we have $(n-k+1)$ items to choose from. Hence in total we have $\frac{n!}{(n-k)!}$ total ways to place ~~n~~ items in k spots. Then because we don't care about the order of the k items, we have to divide by $k!$ to get rid of the ~~ext~~ extra. Hence $\frac{n!}{k!(n-k)!}$ represents how many ways we can choose k items out of n items.

3) Out of n times we need k heads. Assume we have n trials. ~~as~~ Let's say the first k coins turn heads and the last $(n-k)$ toss turn tails. The chance of the happening is $p^k(1-p)^{n-k}$. Now let's find another such event where the first toss is tails, the next k toss are heads, and the last $(n-k+1)$ toss are tails. The probability of this is $(1-p)p^k(1-p)^{n-k+1}$ $= p^k(1-p)^{n-k}$. As we can see out of the n spots ~~we~~ the number of ways we can choose k heads is $\binom{n}{k}$ and the probability of each such orientation happening is $p^k(1-p)^{n-k}$, hence the probability of getting k heads in n tosses is $\binom{n}{k} p^k (1-p)^{n-k}$

4) Let $V = \int_{-\infty}^{\infty} e^{-x^2/2}\,dx$

$$V^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}\,dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy$$

$r^2 = x^2+y^2$ ~~and~~

$$V^2 = \int_0^{\infty}\int_0^{2\pi} re^{-r^2/2}\,d\theta\,dr = \int_0^{\infty} re^{-r^2/2}(2\pi)\,dr = 2\pi\left[-e^{-r^2/2}\Big|_0^{\infty}\right]$$

$$= 2\pi\left[-e^{0} - e^{0}\right] = 2\pi$$

$$V = \sqrt{2\pi}$$

Hence $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx = \frac{1}{\sqrt{2\pi}}\cdot C = \frac{C}{C} = \underline{1}$