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Hw 1a

1. $P(X \geq 1) = 1 - \left(\frac{5}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^1 = 1 - \frac{25}{36} = \boxed{\frac{11}{36}}$

$P(x \geq 1) = 1 - \left(\frac{5}{6}\right)^{10} = \boxed{.838}$

$P(x \geq 1) = 1 - \left(\frac{5}{6}\right)^n$

2. n choose k: Picking k elements from a set n. There are n ways to pick the first, n-1 to pick the second and so on. This means, in an ordered list, we have

$n(n-1)(n-2) \ldots (n-k+1) = \frac{n!}{(n-k)!}$

$$\frac{\frac{n!}{(n-k)!}}{k!}$$

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3. Proving Binomial distribution $\binom{n}{k} p^k (1-p)^{n-k}$

If we throw a loaded coin n times, the probability that the first k tosses land on heads is given by $\underbrace{(p \cdot p \ldots)}_{k} \cdot \underbrace{(1-p)(1-p)\ldots(1-p)}_{n-k}$ Clearly, our two outcomes are covered

so $\binom{n}{k} p^k (1-p)^{n-k}$ covers all the cases of getting k heads in n tosses.

4. $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = 1$

$b^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2}\, dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}\, dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{(x^2+y^2)}{2}}\, dx\, dy$

We will use polar coordinates, so: $b^2 = \int_0^{\infty}\int_0^{2\pi} r \cdot e^{-\frac{r^2}{2}}\, d\theta\, dr$

$u = -\frac{r^2}{2}$ $\quad = \int_0^{\infty} re^{-r^2/2}\left(\int_0^{2\pi} d\theta\right) dr$ $\quad \int e^u = e^u$

$du = r$

$b^2 = 2\pi \int_0^{\infty} re^{-r^2/2}\, dr = \int e^u\, du = e^{-r^2/2}\Big|_0^{\infty} = 0 + 1 = \boxed{1}$

$b^2 = 1 \rightarrow \boxed{b = 1}$