Homework 19

1. The sample space are

{1, 1} {1, 2} {1, 3}, {1, 4} {1, 5} {1, 6}
{2, 1} {2, 2} {2, 3} {2, 4} {2, 5} {2, 6}
{3, 1} {3, 2} {3, 3} {3, 4} {3, 5} {3, 6}
{4, 1}, {4, 2} {4, 3} {4, 4} {4, 5} {4, 6}
{5, 1}, {5, 2} {5, 3} {5, 4} {5, 5} {5, 6}
{6, 1} {6, 2} {6, 3} {6, 4} {6, 5} {6, 6}

(a) At least one of the shows 1

$P = (11/36)^2 = \cancel{12}1/108$

(b) $P = (11/36)^{10} = \left(\frac{11}{36}\right)^{10}$

(c) $P = (11/36)^n$

2. $\frac{n!}{k!(n-k)!}$ = binomial (n, k)

expand $(1+x)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$f(x) = f(0) + f'(0)\cdot x + f''(0)\cdot x^2/2! + \dots$

$f(x) = (1+x)^n$

$f(0) = 1$

$f'(x) = n\cdot(1+x)^{n-1}$; $f'(0) = n$

$f''(x) = n\cdot(n-1)\cdot(1+x)^{n-2}$; $f''(0) = n\cdot(n-1)$

$f^k(x) = n\cdot(n-1)\cdot(n-2)\dots(n-k+1)\cdot(1+x)^{n-k}$, $f^{(k)}(0) = n\cdot(n-1)\dots(n-k+1)$

$(1+x)^n = 1 + n\cdot x + n\cdot(n-1)/2!\; x^2 + n\cdot(n-1)\cdot(n-2)/3!\; x^3 \dots n\cdot(n-1)\cdot(n-k+1)/$

binom$(n, k) = n\cdot(n-1)\dots(n-k+1)/k!$

3. For the coin we are given here that:

P(head) = p and therefore P(tail) = 1 - p

So the number of getting head $N \sim Bin(N, p=q)$

The probability of getting k head is

$P = p^k (1-p)^{N-k}$

$P(N=k) = \binom{N}{k} p^k (1-p)^{N-k}$

4. $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = 1$

$c = \int_{-\infty}^{\infty} e^{-y^2/2} dy$

$c^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2} dy\right)$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2} dx dy$

Because x^2+y^2 becomes r^2, $0 \le r < \infty$, $0 \le \theta < 2\pi$

$c^2 = \int_0^{\infty}\int_0^{2\pi} r e^{-r^2/2} d\theta dr$

$= 2\pi \int_0^{\infty} r e^{-r^2/2} dr$

$= \int_0^{\infty} r e^{-r^2/2} d\theta dr$

$= -e^{-r^2/2}\Big|_0^{\infty}$

$= 1$