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1. Tetrahedro : $F:4$; $V:4$; $E:6$
   Cube : $F:6$; $V:8$; $E:12$
   Octahedron : $F:8$; $V:6$; $E:12$
   Dodecahedron : $F:12$; $V:12$; $E:30$
   Icosahedron : $F:20$; $V:12$; $E:30$

2. If the graph is not a tree, it has at least one cycle. Removing any edge of that cycle, decrease both e and f, but keeps v constant, in other words $e'=e-1$, $f'=f-1$, $v'=v$.
So $v-e+f=v'-e'+f'$ and by the inductive hypothesis is 1.
The initial base case is a tree. A tree is a connected graph without cycles. The umber of regions, f is not 0. Of course it must have at least one vertex of degree 1, it is called a leaf. Removing such a leaf reduces the number of vertices by one, and also the number of edges by 1, so we have $f'=0$. $v'=v-1$, $e'=e-1$.
So once again $v-e+f+v'-e'+f'$, and this equals 1 by the inductive hypothesis.

3. (i) Since every edges belongs to two vertices.
then $2E=aV$
And since every face has b edges coming out of it,
then $2E=bF$.
So $V=\frac{2E}{a}$, $F=\frac{2E}{b}$.
Corresponding Platonic solid, for example, Tetrahedro, there are 3 edges meeting every vertex, it has 4 vertices and 6 edges. plug in $V=\frac{2E}{a}$, which is true.

(ii) $\frac{2E}{a} - E + \frac{2E}{b} = 2 \Rightarrow E\left(\frac{2}{a} + \frac{2}{b} - 1\right) = 2 \Rightarrow$

Corresponding Platonic solid, for example, Tetrahedro.
there are 3 edges meeting every vertex, there are 3 edges around every face, so $a=3$, $b=3$, it has 4 vertices and 6 edges.
plug in $E = \frac{2}{\frac{2}{a}+\frac{2}{b}-1}$, $E = \frac{2}{\frac{2}{3}+\frac{2}{3}-1} = 6$. which is true.

(iii) when $a=3, b=3$, $E = \frac{2}{\frac{2}{3}+\frac{2}{3}-1} = 6$, it is a tetrahedro

When $a=3$, $b=4$, $E = \frac{2}{\frac{2}{3}+\frac{2}{4}-1} = 12$, it is a cube.

When $a=4$, $b=3$, $E = 12$, it is a octahedron.

When $a=3$, $b=5$, $E = \frac{2}{\frac{2}{3}+\frac{2}{5}-1} = 30$, it is a dodecahedron

When $a=5$, $b=3$, $E = 30$, it is an icosahedron.

When $a=4$, $b=5$, $E = \frac{2}{\frac{2}{4}+\frac{2}{5}-1} = -20$ X

When $a=5$, $b=5$, $E = \frac{2}{\frac{2}{5}+\frac{2}{5}-1}$ X

When $a=5$, $b=4$, X

4. A perfect solid means that the faces are congruent and the same number meet at each vertex. But the surface of a soccer ball is made up of pentagons and hexagons, which doesn4 not satisfy the defination of a perfect solid.

Soccer ball: $F: 32$, $E=90$, $V=60$.

$V - E + F = 60 - 90 + 32 = 2$, which is also valid for a soccer ball.