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1. Tetrahedron: $V=4$, $E=6$, $F=4$
Cube: $V=8$, $E=12$, $F=6$
Octahedron: $V=6$, $E=12$, $F=8$
Dodecahedron: $V=20$, $E=30$, $F=12$
Icosahedron: $V=12$, $E=30$, $F=20$

2. Start off with a planar map, without counting the "missing side". then, $V-E+F=1$. Begin removing edges one at a time. Each time an edge is removed, the number of edges and the number of faces each decreases by 1, but the number of vertices remains the same. So, $V'=V$, $E'=E-1$, $F'=F-1$. So, $V'-E'+F' = V-E+1+F-1 = V-E+F=1$ still holds.
Eventually, you get a tree, where there are no faces, so $F=0$, and only vertices and edges. Start removing leaves - vertices of degree 1 - one at a time until only one vertex remains. Then, $V=1$, $E=0$, and $F=0$, so $V-E+F = 1-0+0=1$.
Since it holds for all cases, then $V-E+F=1$ must always be true.

3. (i) Since every edge connects exactly two vertices and each vertex has a edges coming out of it, then $2E = aV$. So, $V=\frac{2E}{a}$.
Since every edge connects two faces and the number of edges around every face is b, $2E = bF$. So, $F=\frac{2E}{b}$

(ii) $V-E+F=2$ so $\frac{2E}{a} - E + \frac{2E}{b} = 2$ $E\left(\frac{2}{a}-1+\frac{2}{b}\right)=2$
so $E = 2/\left(\frac{2}{a}-1+\frac{2}{b}\right)$

(iii) $a=3$, $b=3$ $E = 2/\left(\frac{2}{3}-1+\frac{2}{3}\right) = 2/\frac{1}{3} = 6$ ✓
$a=3$, $b=4$ $E = 2/\left(\frac{2}{3}-1+\frac{1}{2}\right) = 2/\frac{1}{6} = 12$ ✓
$a=3$, $b=5$ $E = 2/\left(\frac{2}{3}-1+\frac{2}{5}\right) = 2/\frac{1}{15} = 30$ ✓
$a=4$, $b=3$ $E = 2/\frac{1}{2}-1+\frac{2}{3} = 2/\frac{1}{6} = 12$ ✓
$a=4$, $b=4$ $E = 2/\frac{1}{2}-1+\frac{1}{2} \neq$ a positive number
$a=4$, $b=5$ $E = 2/\frac{1}{2}-1+\frac{2}{5} \neq$ a positive integer

$a=5, b=3 \quad E = \dfrac{2}{\frac{2}{5} - 1 + \frac{2}{3}} = \dfrac{2}{\frac{1}{15}} = 30$ ✓

$a=5, b=4 \quad E = \dfrac{2}{\frac{2}{5} - 1 + \frac{1}{2}} \neq$ a positive integer

$a=5, b=5 \quad E = \dfrac{2}{\frac{2}{5} - 1 + \frac{2}{5}} \neq$ a positive integer

4. A soccer ball is made of pentagons and hexagons for the faces, so not all of the faces are the same. So it is not a perfect or platonic solid.

Soccer ball: $V = 60$, $E = 90$, $F = 20$ hexagons $+ 12$ pentagons

$60 - 90 + 32 = -30 + 32 = 2$

So Euler's formula is valid for a soccer ball.