MATH 437 : NINA CHALGERI HW 18 * YOU CAN POST

①

NAME	# of Vertices	# of EDGES	# of FACES
Tetrahedron	4	6	4
Cube	8	12	6
Octahedron	6	12	8
Dodecahedron	20	30	12
Icosahedron	12	30	20

② Induction on the number of edges

- If the graph is not a tree, it has at least one cycle
- So when an edge is removed, v stays constant but e and f decrease by 1:

$$e' = e - 1 \qquad f' = f - 1 \qquad v' = v$$

$\Rightarrow v - e + f = v' - e' + f'$ and this equal 1 by the induction hypothesis

- Base Case is a tree so there is only one face, this is the
- "infinite ocean" so if we do not count this, then $f' = 0$
- Then remove the leaf, which is a vertex of degree 1
- This means that both v and e go down by 1 so:

$$f' = 0 \qquad v' = v - 1 \qquad e' = e - 1$$

- Therefore, $v - e + f = v' - e' + f' = 1$ by the inductive hypothesis
- The ultimate base case: one vertex and no regions:

$$v = 1 \qquad e = 0 \qquad f = 0$$

$$\Rightarrow v - e + f = 1 \quad \blacksquare$$

③ (i) [PART (i) ON LAST PAGE]

(ii) $V - E + F = 2$

$$\frac{2E}{a} - E + \frac{2E}{b} = 2$$

$$E\left[\frac{2}{a} - 1 + \frac{2}{b}\right] = 2$$

$$\frac{2}{a} - 1 + \frac{2}{b} = \frac{2}{E}$$

$$\frac{2b}{ab} - \frac{ab}{ab} + \frac{2a}{ab} = \frac{2}{E}$$

$$\frac{2b - ab + 2a}{ab} = \frac{2}{E}$$

$$\Rightarrow \boxed{E = \frac{2ab}{2b - ab + 2a}}$$

(iii) (a, b)

(a,b)	E	SOLID			
(3, 3) ⇒	6	[TETRAHEDRON] ~~[CUBE]~~	V = 4	E = 6	F = 4
(3, 4) ⇒	12	[CUBE]	V = 8	E = 12	F = 6
(3, 5) ⇒	30	[DODECAHEDRON]	V = 20	E = 30	F = 12
(4, 3) ⇒	12	[OCTAHEDRON]	V = 6	E = 12	F = 8
(5, 3) ⇒	30	[ICOSAHEDRON]	V = 12	E = 30	F = 20

④ A soccer ball is not a platonic solid because it has 12 pentagons and 20 hexagons which means the faces are not congruent.

$$F = 12 + 20 = 32 \quad V = 60$$

$$\Downarrow$$

$$V - E + F = 2$$
$$60 - E + 32 = 2$$
$$\Rightarrow E = 90$$

$\rightarrow V = 60 \quad E = 90 \quad F = 32$

③ (i) PROVE $V = \frac{2E}{a}$

- ~~Since~~ V is the total number of ~~edges~~ vertices and a is the number of edges meeting each vertex (the degree), ~~then the~~ ~~[illegible]~~
- We can rewrite the formula as:

$$aV = 2E$$

- An edge is a segment that connects two vertices which means that one edge reaches two vertices.
- This tells us that:

$$\frac{aV}{2} = E$$

which means the number of edges is half of ~~the~~ aV since each edge connects two vertices. ■

PROVE $F = \frac{2E}{b}$

- b is the number of edges around every face. So if we rewrite:

$$bF = 2E$$

- This tells us that the number of edges around every face times the number of faces is equal to 2 times the number of edges.
- Every edge of a face is also the edge of a different face.
- This means that every one edge is part of the construction for two different faces. ■