Homework 18

① Tetrahedron: 4 V, 6 E, 4 F
Cube: 8 V, 12 E, 6 F
Octahedron: 6 V, 12 E, 8 F
Dodecahedron: 20 V, 30 E, 12 F
Icosahedron: 12 V, 30 E, 20 F

② A non-tree graph has atleast 1 cycle. removing an edge keeps V the same, but $e = e - 1$, $f = f - 1$.

$V - e + f = 1$

A Base case: tree. one face, the infinite ocean. so the new $f = 0$. removing the leaf is a vertex of degree 1, so $v = v - 1$, $e = e - 1$.

$V - e + f = 1$ again.

Another base case: one vertex, no regions

$V = 1, e = 0, f = 0$

$v - e + f = 1$

③ (i) I'm not sure

(ii) $\frac{2E}{a} - E + \frac{2E}{b} = 2$

$E\left(\frac{2}{a} - 1 + \frac{2}{b}\right) = 2$

$\frac{2}{a} - 1 + \frac{2}{b} = \frac{2}{E} \rightarrow \frac{2b - ab + 2a}{ab} = \frac{2}{E}$

$E = \frac{2ab}{2b - ab + 2a}$

(iii) (3,3), V=4, E=6, F=4 (tetrahedron)
(3,4), V=8, E=12, F=6 (cube)
(3,5), V=20, E=30, F=12 (dodecahedron)
(4,3), V=6, E=12, F=8 (octahedron)
(5,3), V=12, E=30, F=20 (icosahedron)

(4) 20 hexagons, 12 pentagons (non convergent faces)

F = 12 + 20 = 32
V = 60

60 − E + 32 = 2 → V = 60
F = 90 E = 90
F = 32