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Yes

H.W 18

1. Tetrahedron: $V=4, F=4, E=6$
hexahedron: $V=8, F=6, E=12$
Octahedron: $V=6, F=8, E=12$
Dodecahedron: $V=20, F=12, E=30$
ISoSahedron: $V=12, F=20, E=30$

2. Suppose that it is not a tree meaning the planar graph has regions so there a cycles. Pick any edge along the cycle and delete it to get a smaller graph and let $v', e', f'$ be the vertices, edges, and faces respectively of the new graph but we notice that when you remove an edge, you take away an edge and a face but the vertices still stay the same so $v' = v$, $e' = e-1$, $f' = f-1$.
We can show a base case of a tetrahedron on a planar graph where $V=4, E=6, F=3$
so $v-e+f=1$ so assume $v-e+f=1$ for some polyhedron on a planar graph. So now we have
$v-e+f = v'-(e'+1)+(f'+1) = v'-e'+f' = 1$ for some polyhedron on a planar graph.
You keep on taking away edges until you get that $v, e, f=0$ now if you take away an edge you are left with $v'=v-1, e'=e-1, f'=0$
and we see again that $v-e+f=(v'+1)-(e'+1)+f' = v'-e'+f' = 1$
and eventually we get to the trivial case that $v=1, e=0, f=0$
so $v-e+f = 1-0-0$ thus $v-e+f=1$

Scanned with CamScanner

3I. Every edge belongs to two vertices
So $2 \cdot E = a \cdot V$ and $2 \cdot E = bF$
Which implies that $V = \frac{2E}{a}$ and $F = \frac{2E}{b}$

3II. Using $V - E + F = 2$,
we get $\frac{2E}{a} - E + \frac{2E}{b} = 2 \rightarrow E\left(\frac{2}{a} - 1 + \frac{2}{b}\right) = 2$

$\rightarrow E = \frac{2}{\frac{2}{a} - 1 + \frac{2}{b}}$

3III). $a = 3, b = 3$ $E = 6$, Tetrahedron
$a = 3, b = 4$ $E = 12, V = \frac{2(12)}{3} = 8$ hexahedron

$a = 4, b = 3$ $E = 12, V = 6$ octahedron
$a = 3, b = 5$ $E = 30, v = 20$ dodecahedron
$a = 5, b = 3$ $E = 30, V = 12,$ Icosahedron

4. A soccer ball is a sphere with n faces as $n \to \infty$ and is not perfect because the sphere is not an infinite faced polyhedron but only has the limit going there.