

1)

Solid	Vertices	Edges	Faces
Tetrahedron	4	6	4
Cube	8	12	6
Octahedron	6	12	8
Dodecahedron	20	30	12
Icosahedron	12	30	20

2)

We say that

$$e' = e - 1, f' = f - 1, v' = v$$

$$v - e + f = v' - e' + f'$$

F cannot be 0, thus must me at least 1 vertex.

$$v - e - 1 + f - 1 + v - e + f = 1$$

3)

a)

If we have a cube with $V=8$, $E=12$, $F=4$, and using a and b for vertices connected by edges and edges around each face, we can determine a relation for V and F using E , a , and b .

Since this is a perfect platonic solid, we can see that each vertex has 3 edges(a) and each face has 4 edges adjacent(b). The total amount of vertexes can then be equated by the amount total of edges and the number of edges per vertex, and since each edge connects 2 vertexes, we can say that $2 \cdot E/a$ to get the total amount of vertexes on the solid, netting us 8 vertexes. Using the same logic, if we know the total amount of edges touching each face and the total amount of edges, and since each edge will be touching 2 different faces, $2 \cdot E/b$ will net us 6 faces.

b)

by using the equations above, and the formula $V-E+F=2$, we can combine and make this equation in terms of E :

$$\frac{2E}{a} - E + \frac{2E}{b} = 2$$

$$E \left(\frac{2}{a} - 1 + \frac{2}{b} \right) = 2$$

$$E \left(\frac{2b + 2a - ab}{ab} \right) = 2$$

$$E = \frac{2ab}{2b + 2a - ab}$$

c)

for $a=3$, $b=5,4,3$ the corresponding shapes are dodecahedron, cube, and the tetrahedron.

For $a=4, b=3$, we have a octahedron.

For $a=5, b=3$, we have a icosahedron.

4) a soccer ball is truncated which means it is not the full shape, where it is partly cut off to make a ball.

$$V=60, F=32, E=90$$

$$60-90+32=2$$