History of Math | Homework 18 | Karan Amin

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1) Tetrahedon : $V=4$, $E=6$, $F=4$

Cube: $V=8$, $E=12$, $F=6$

Octahedron: $V=6$, $E=12$, $F=8$

Dodecahedron: $V=20$, $E=30$, $F=12$

Icosahedron: $V=12$, $E=30$, $F=20$

2) Induction on number of edges

~~If the graph is a tree:~~

~~Base: $e=1$, then we~~ have

Base case: $e=0$, which means we simply have a vertex, and no face, therefore $v=1$, $e=0$, $f=0$ $\rightarrow$ $v-e+f=1$

Assume $v-e+f=1$ for some graph with n edges. Then either the graph is a tree or not.

If it's a tree, then after removing an edge, we lose an edge, and a vertex but keep the total number of faces at 0.

Hence $f'=f=0$, $e'=e-1$, $v'=v-1$, therefore

$v'-e'+f' = (v-1)-(e-1)+f = v-e+f=1$ by the inductive hypothesis. Hence the equation is true for the new graph.

If the new graph is not a tree and we remove an edge in a cycle, then $v'=v$, $e'=e-1$, $f'=f-1$, then

$v'-e'+f' = v-(e-1)+(f-1) = v-e+f=1$ by the I.H.

Hence the equation $v-e+f=1$ is true for any planer graph.

3)

3i) Assume a platonic solid has V vertexes, E edges, and F faces, a edges meeting every vertex, and b edges around every face.

Because ~~we~~ every vertex has the same number of edges meeting it, if we multiply aV, then we get the total number of edges counting the number of edges at each vertex for all vertexes. Since each edge coming out of a vertex meets another vertex, we double counted. Hence $\frac{aV}{2}$ represents the total number of ~~vertexes~~ edges E. Therefore

$$\frac{aV}{2} = E \rightarrow V = \frac{2E}{a}$$

Similarily $\frac{bF}{2}$ represents the total number of edges E since bF counts the total number of edges around each face for all faces, we double counted, and since ~~we~~ each edge joins two faces ~~Because~~ Then therefor $\frac{bF}{2}$ represents the total number of edges E.

$$\frac{bF}{2} = E \rightarrow F = \frac{2E}{b}$$

3ii) We know $V = \frac{2E}{a}$, $F = \frac{2E}{b}$, then $V - E + F = \frac{2E}{a} - E + \frac{2E}{b} = 1$

$$\frac{b2E}{ab} - \frac{abE}{ab} + \frac{2aE}{ab} = \frac{2bE - abE + 2aE}{ab} = \frac{E(2b - ab + 2a)}{ab} = 1$$

Then $\boxed{E = \frac{ab}{2b - ab + 2a}}$

3iii) Tetrahedron $[a, b] = [3, 3]$ $V =$

$$E = \frac{3 \cdot 3 \cdot 2}{2 \cdot 3 - 3 \cdot 3 + 2 \cdot 3} = \frac{9 \cdot 2}{6 - 9 + 6} = \frac{9 \cdot 2}{3} = 6$$

$[V, E, F] = [4, 6, 4]$

Cube $[a, b] = [3, 4]$

$$E = \frac{12 \cdot 2}{6 - 12 + 8} = \frac{24}{2} = 12$$

$[V, E, F] = [8, 12, 6]$

Octahedron: $[a, b] = [4, 3]$

$$E = \frac{12-2}{8-12+6} = \frac{24}{2} = 12$$

$[V, E, F] = [6, 12, 8]$

Dodecahedron $[a, b] = [3, 5]$

$$E = \frac{15-2}{6-15+10} = \frac{\cancel{45}\ 30}{1} = 30$$

$[V, E, F] = [6, 12, 8]$

Icosahedron $[a, b] = [5, 3]$

$$E = \frac{30}{10-15+6} = 30$$

$[V, E, F] = [12, 30, 20]$

4) A soccer ball has 12 pentogons and 20 hexagons hence ~~a~~ not all faces have the same number of edges around it, therefore cannot be a platonic solid

On a soccer ball $V = \cancel{12}\ 60$, $E = 90$, $F = 32$ hence

$$V - E + F = 60 - 90 + 32 = 92 - 90 = 2$$