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Hw18.

1. Tetrahedron – 4 vertices, 6 edges, 4 faces
   Cube – 8 vertices, 12 edges, 6 faces
   Octahedron – 6 vertices, 12 edges, 8 faces
   Icosahedron – 12 vertices, 30 edges, 20 faces

2. This requires a proof by induction:

Assume our graph is <u>not</u> a tree, then there must be at least one cycle. If we remove any edge along the cycle, we get a smaller graph where the number of edges and faces are both decreasing by 1. If we say for our new graph that $v' = v$, $e' = e-1$, $f' = f-1$

<u>Then</u>, $v - e + f = v' - e' + f'$ → we have shown our base case with the inductive hypothesis.

Consider a tree, $f = 0$, there must be at least 1 leaf → a vertex of degree 1.

Taking that leaf away gives, $f' = 0$, $v' = v-1$, $e' = e-1$ so, $v - e + f = v' - e' + f'$

Our <u>last</u> case to consider is where $v = 1, e = 0, f = 0$. In this case $v - e + f = 1$. We have shown that for trees and maps $v - e + f = 1$.

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3.

i) $V = \frac{2E}{a}$, $F = \frac{2E}{b}$

If every vertex has a edges attached to it, and V vertices, there must be V edges. All edges touch two vertices so $2E = aV \rightarrow E = \frac{aV}{2}$, thus $V = \frac{2E}{a}$.

Now, let's say there are b edges at each face so $bf = 2E$

Thus, $F = \frac{2E}{b}$

ii) $V - E + F = 2 \rightarrow$ So, $\frac{2E}{a} - E + \frac{2E}{b} = 2 \rightarrow \frac{2E}{a} + \frac{2E}{b} - E = 2 \rightarrow E\left(\frac{2}{a} + \frac{2}{b} - 1\right) = 2$

$$E = \frac{2}{\frac{2}{a} + \frac{2}{b} - 1}$$

iii) $a \geq 3$, $b \leq 5$

$a=3, b=3$ $[V=4, E=6, F=4]$ $\frac{12}{2} = \boxed{6}$ → Tetrahedron

$a=3, b=4$ $[V=8, E=12, F=6]$, $\frac{24}{2} = \boxed{12}$ → Hexahedron

$a=3, b=5$ $[V=20, E=30, F=12]$, $\frac{30}{1} = \boxed{30}$ Dodecahedron

$a=4, b=3$, $[V=6, E=12, F=8]$, $\frac{40}{2} = \boxed{20}$ Octahedron

$a=5, b=3$ $[V=12, E=30, F=20]$ $= \frac{30}{1} = \boxed{30}$ Icosahedron?

4. A Soccer ball is not perfect because its faces are made up of hexagons + pentagons which are not the same shape.

$[V=60, E=90, F=32]$

$\boxed{V - E + F = 2}$