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Homework 18 11/21/2021

1.

Platonic solids	V	E	F
Tetrahedron	4	6	4
Cube	8	12	6
Octahedron	6	12	8
Dodecahedron	20	30	12
Icosahedron	12	30	20

2. Proof that for a planar map, $V-E+F=1$:
By induction on the number of edges.
If the map is not a tree, then it has at least one cycle. Then we can remove any edge that is part of a cycle, such that the number of vertices remains the same.
Take the initial number of vertices, edges and faces to be v, e, f, respectively. Then after we remove an edge:

$$v' = v, \quad e' = e-1, \quad f' = f-1$$

Then $v-e+f = v'-e'+f'$. Once we apply the inductive hypothesis, we will see that equals 1.
To prove the initial base case, consider a tree because it doesn't have any cycles and is a connected graph so $f>0$. A tree must have at least 1 leaf - a vertex of degree 1.

Removing this leaf gives:

$$f' = 0, \quad v' = v - 1, \quad e' = e - 1.$$

Now we have: $v - e + f = v' - e' + f'$.

The ultimate base case is: $v = 1, e = 0, f = 0$ and here $v - e + f = 1$. Therefore, for trees and maps it holds that $v - e + f = 1$.

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3 a) In a platonic solid, each edge touches 2 faces, so if there are b edges around each face then $b \cdot F$ will count all the edges twice. Thus we have that $b \cdot F = 2E$, where E is the total number of edges and:

$$\boxed{F = \frac{2E}{b}.}$$

At the same time each edge touches 2 vertices, so if there are a edges at every vertex then $a \cdot V$ will count all the edges twice. Thus, we have that $a \cdot V = 2E$ and

$$\boxed{V = \frac{2E}{a}}$$

b) Use the above to plug into $V - E + F = 2$:

$$\frac{2E}{a} - E + \frac{2E}{b} = 2.$$

$$E\left(\frac{2}{a} - 1 + \frac{2}{b}\right) = E\left(\frac{2b - ab + 2a}{ab}\right) = 2$$

$$E = \frac{2ab}{2b + 2a - ab}$$

c) $3 \le a, b \le 5$ that makes E positive integers.

(a, b)	E
$(3,3)$	$E = \frac{2\cdot3\cdot3}{2\cdot3+2\cdot3-3\cdot3} = \frac{18}{12-9} = 6$
$(3,4)$	$E = \frac{2\cdot3\cdot4}{2\cdot3+2\cdot4-3\cdot4} = \frac{24}{14-12} = 12$
$(3,5)$	$E = \frac{2\cdot3\cdot5}{2\cdot3+2\cdot5-3\cdot5} = \frac{30}{16-15} = 30$
$(4,5)$	$E = \frac{2\cdot4\cdot5}{2\cdot4+2\cdot5-4\cdot5} = \frac{40}{18-20} = \frac{40}{-2} = -20$

Platonic solids:

$(3,3) \rightsquigarrow \quad E=6, \quad V=4, \quad F=4$

$(3,4) \rightsquigarrow \quad E=12 \quad V=8 \quad F=6$

$(3,5) \rightsquigarrow \quad E=30 \quad V=20 \quad F=12$

4. A soccer ball is not perfect because it is not made up of the same shapes. It has 12 pentagons and 20 hexagons.
A soccer ball has:
32 faces, 60 vertices and 90 edges.
So Euler's formule
$V-E+F = 60-90+32=2$ still holds for soccer ball.