HW 18 ok to post

1) Tetrahedron → $V=4$, $F=4$, $E=6$
Cube → $V=8$, $F=6$, $E=12$
Octahedron → $V=6$, $F=8$, $E=12$
Dodecahedron → $V=20$, $F=12$, $E=30$
Isocahedron → $V=12$, $F=20$, $E=30$

2) Remove one face of the polyhedral surface. By pulling the edges of points and curves in such a way that the perimeter of the missing face is placed externally surrounding the graph obtained. After this deformation, the regular faces are generally not regular anymore. The # of vertices & edges has remained the same, but the number of faces has been reduced by 1. Therefore proving Euler's formula for the polyhedron reduces to proving $V-E+F=1$ for this deformed planar object.

3) # edges meeting every vertex 'a', # edges around face 'b'

(i) $aF = 2E = bV$

$V-E+F=2$

$$E = \frac{2ab}{4-(a-2)(b-2)}$$

$$V = \frac{4a}{4-(a-2)(b-2)} \rightarrow V = \frac{2E}{a}$$

$$F = \frac{4b}{4-(a-2)(b-2)} \rightarrow F = \frac{2E}{b}$$

ii) $V - E + F = 2$

$V = \frac{4a}{4-(a-2)(b-2)}$ $E = \frac{2ab}{4-(a-2)(b-2)}$

$F = \frac{4b}{4-(a-2)(b-2)}$

iii) $a = 3$ $b = 5$

$E = \frac{2(3)(5)}{4-(3-2)(5-2)} = \frac{30}{1} = \underline{30}$

All of them are positive if conditions of a & b are met

4) The soccer ball is not perfect because it is not perfectly round. It is a truncated icosahedron. It has 32 faces, 90 edges, and 60 vertices.

Euler's formula is valid as:

$60 - 90 + 32 = 2$ ✓