Homework 18

1. Tetrahedron, Cube, Octahedron, Icosahedron, Dodecahedron

2. $V - E + F = 1$

Proof: If the graph has at least one cycle, Removing any edge of that cycle, decreases both e and f, but keeps v constant

$E' = E - 1,\ f' = F - 1,\ V' = V$

So $V - E + F = V' - E' + F' = 1$

It it is a tree, connected without cycles. F is not 0

$F' = 0,\ V' = V - 1,\ E' = E - 1$

So once again $V - E + F + V' - E' + F'$ Equal to 1.

Thus proved

3. $V = 4 \qquad a = 3$

$E = 6 \qquad b = 3$

$F = 4$

Thus, $V = 4 = \frac{2(6)}{3} = 4$

$F = 4 = 2(6)/3 = 4$

Similar proof for solids

$V - E + F = 2 \qquad V = 2E/a$

Substitute $\qquad F = 2E/b$

$\frac{2E}{a} - E + \frac{2E}{b} = 2 \qquad E = \frac{2ab}{2a + 2b - ab}$

$E(2/a + 2/b - 1) = 2$

$a = 3,\ b = 3,\ E = 6 \rightarrow$ Tetrahedron $V = 4,\ F = 4$

$a = 3 \quad b = 4,\ E = 12 \rightarrow$ octahedron $V = 8,\ F = 6$

$a = 4,\ b = 3,\ E = 12, \rightarrow$ octahedron

4. It is not a perfect round

A soccer ball has 12 pentagonal faces and 20 hexagonal faces

$E = \underline{12\cdot 5+20\cdot 6} = 90$

To find the number of V

$F - E + V = 2$

$F = 22$, $E = 90$

$V = 70$

Euler's formula relating V, E and F is also valid for a soccer ball