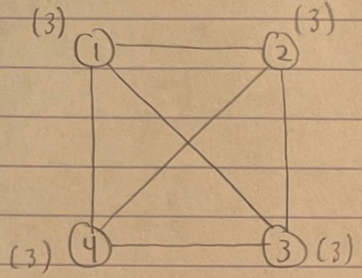
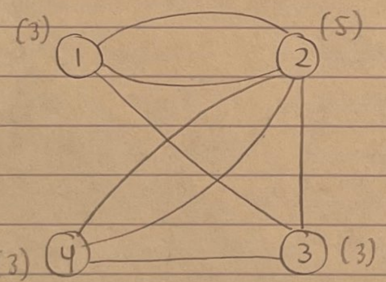
Sarah Magno
Dr. Z, History of Math
11/14/21

Homework for Lecture 17 - OK to post

1



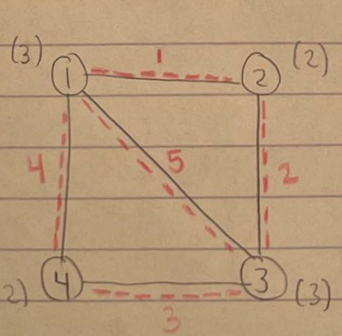
$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$



$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{pmatrix} \end{array}$$

Both of these graphs have neither a Eulerian path nor a Eulerian cycle because all of the vertices have odd degree.

2



$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

= example of a Eulerian path

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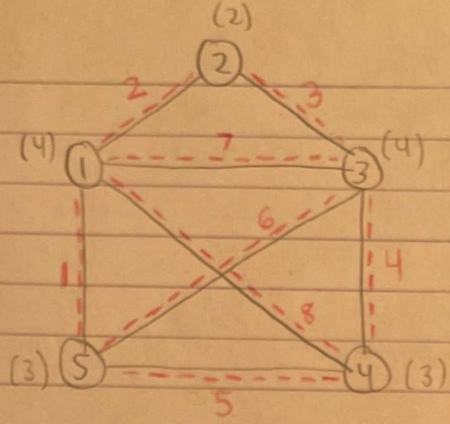
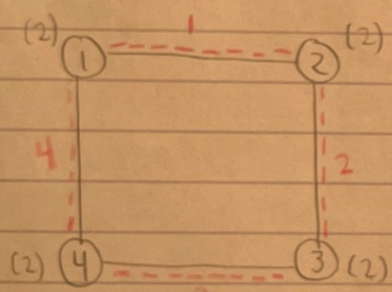


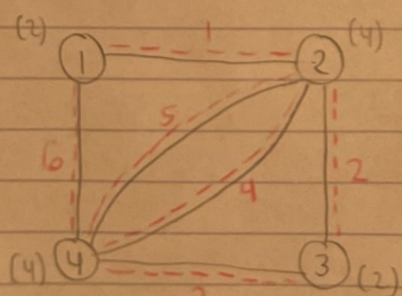
$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 & 0 & 1 \\ 5 & 1 & 0 & 1 & 1 & 0 \end{array}$$

Both of these graphs have a Eulerian path since all but two vertices are odd.

(3)



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 & 0 \end{array}$$



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 2 & 1 & 0 \end{array}$$

Both of these graphs have Eulerian cycle because all their degrees are even.

= example of Eulerian cycle

(4) In a Eulerian cycle, each edge is visited exactly once and the route starts and ends at the same vertex. Suppose that the starting vertex is called a. When the route leaves a, that adds 1 to a's degree. When the route goes through any other vertex along the way, it adds 2 to that vertex's degree, since it needs to enter and leave that vertex. When the route comes back to finish at a, it adds one more to a's degree. This shows that the necessary condition for a Eulerian cycle is that all vertices have an even number of edges incident to it (in other words, every vertex's degree is even.)

(5) In a Eulerian path, each edge is visited exactly once, and the route starts and ends at different vertices. Suppose that the starting vertex is called a. When the route leaves a, that adds 1 to a's degree. When the route goes through any other vertex along the way, it adds 2 to that vertex's degree, since it needs to enter and leave that vertex. When the route ends at the finishing vertex, call it b, that adds 1 to b's degree. This shows that the necessary condition for a Eulerian path is that all the vertices except two have even degree, since a and b will have odd degree. Also, we showed that the path started and ended at a and b respectively, and those are the vertices with odd degree.