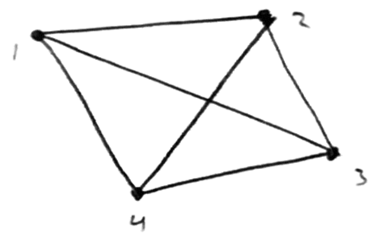
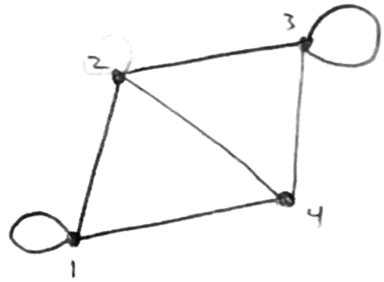
History of Math | Homework 17

①



Has none

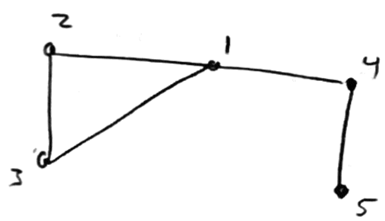


	1	2	3	4
1	0	1	1	1
2	1	0	1	1
3	1	1	0	1
4	1	1	1	0

All vertices have odd degrees [sum of row = degree] hence no path or circuit/cycle

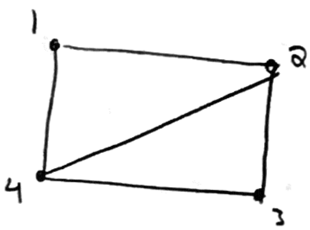
	1	2	3	4
1	1	1	0	1
2	1	0	1	1
3	0	1	1	1
4	1	1	1	0

②



	1	2	3	4	5
1	0	1	1	1	0
2	1	0	1	0	0
3	1	1	0	0	0
4	1	0	0	0	1
5	0	0	0	1	0

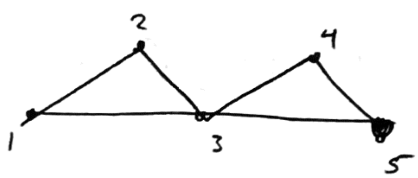
541231 is a Eulerian path that starts at 5 and ends at 1.



	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	0	1	0	1
4	1	1	1	0

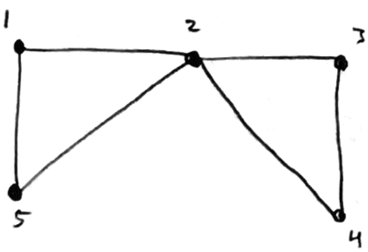
234214 is a eulerian path

③



	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	0	0
3	1	1	0	1	1
4	0	0	1	0	1
5	0	0	1	1	0

1234531 is a eulerian cycle



	1	2	3	4	5
1	0	1	0	0	1
2	0	1	1	1	1
3	0	1	0	1	0
4	0	1	1	0	0
5	1	1	0	0	0

2152342 is a eulerian cycle

4) Assume a graph G has a eulerian circuit/cycle C. Because it is eulerian, it contains every edge and vertex in the graph. For every vertex $v \in G$ we need to show that v has an even degree. First assume v is not the starting and ending vertex in C. Then v must be in the middle of the cycle. Then everytime v is "touched" in C, there are 2 edges, one that goes into v [v is an endpoint of an edge], and one edge that leaves v [v is a startpoint of an edge]. V can be encountered k times, which means there are 2k edges on v, thus has an even degree. The start vertex has an outgoing edge at the begining and one incoming edge at the end of c, hence all vertexes in G are even.

5) Assume graph G has a eulerian path P that starts at u and ends at v. Then in the path P, it leaves u one more time than an edge enters u, and similarly, edges leave v one less time than it enters v. Therefore using a similar argument as above, it must be that u and v have odd degrees, while all other vertexes must be even degrees from an argument similar to 4), hence proved.