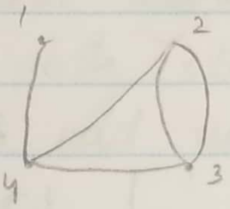
Anna Zhebrun

ok

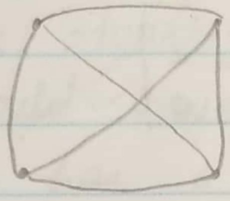
Homework 17. 11/13/2021

1. a)



$$\begin{array}{cc} & \text{\# edges} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} & \begin{matrix} 1 \\ 3 \\ 3 \\ 3 \end{matrix} \end{array}$$

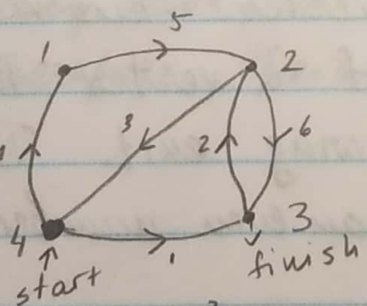
b)



$$\begin{array}{cc} & \text{\# edges} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} & \begin{matrix} 3 \\ 3 \\ 3 \\ 3 \end{matrix} \end{array}$$

Neither of the above examples can have a Eulerian cycle because some of their vertices have odd number of edges, or a Eulerian path because they have more than 2 vertices with odd edges

2. a)

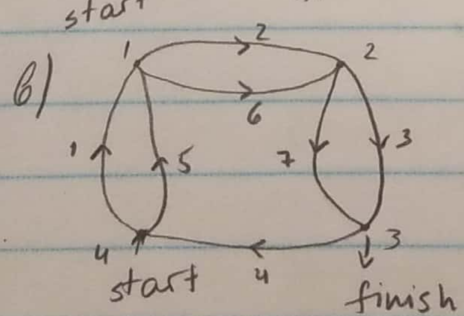


$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Eulerian path

$4 \to 3 \to 2 \to 1 - 4 \to 2 \to 3$

b)

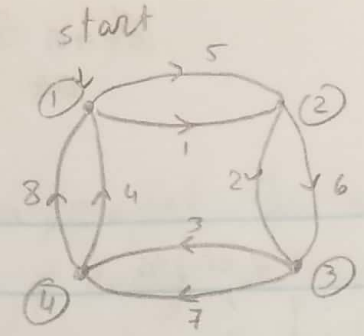


$$\begin{pmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{pmatrix}$$

Eulerian path

$4 \to 1 \to 2 \to 3 \to 4 \to 1 \to 2 \to 3$

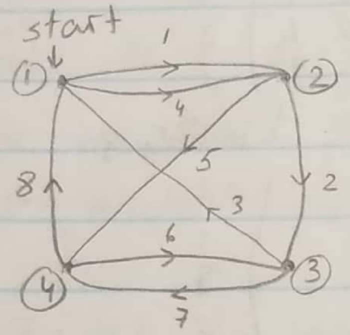
3.



$$\begin{pmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \end{pmatrix}$$

Eulerian cycle

$1 \to 2 \to 3 \to 4 \to 1 \to 2 \to 3 \to 4 \to 1$



$$\begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$

Eulerian cycle:

$1 \to 2 \to 1 \to 2 \to 4 \to 3 \to 4 \to 1$

4. For a Eulerian cycle to exist in a graph, all the vertices must have even number of edges.

Proof:

By definition, a Eulerian cycle traverses all edges exactly once. It also has to start and finish at the same vertex. This means that for every entry of a vertex there is a preceding or following exit. This means that there must be an even number of edges at every vertex.

5. For a Eulerian path to exist every vertex except 2 must have even number of edges, and the path starts and ends at a vertex with odd number of edges.

Proof:

The starting and ending vertices of the graph should have an odd number of edges, because one has a lone entry edge and one has a lone exit edge. If there are more edges, then they come in pairs because that means the vertex is being entered from one side and exits the other.

All the vertices inside should be even because once they are entered through 1 edge, they have to be departed from another. Therefore, their edges come in pairs.